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## **Assortative Matching, Adverse Selection, and Group Lending\***

**Joel M. Guttman**

**Abstract:** This note reconsiders a theoretical result asserted to explain the success of group lending programs in LDCs. It has been claimed that if groups are allowed to form themselves, risky and safe borrowers will sort themselves into relatively homogenous groups. This "positive assortative matching" can be exploited by lenders to solve an adverse selection problem that would otherwise undermine the effectiveness of such lending programs. This note shows that the positive assortative matching result does not necessarily hold if earlier models are extended to incorporate dynamic incentives.

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# Assortative Matching, Adverse Selection, and Group Lending

Joel M. Guttman\*

## 1 Introduction

Group lending is widely regarded as one of the most important institutional innovations in development policy in the last quarter century.<sup>1</sup> In order to explain the success of group lending, which is reflected by repayment rates of 96 to 98 percent in many cases, theorists have focused on three problems of credit provision that are solved better by group lending than by individual lending in the context of small villages in LDCs. The present note reexamines the underpinnings of *one* of these theoretical approaches, and shows that once the models developing this approach are extended in a plausible way, the rationale they provide for group lending becomes problematic.

The three problems of credit provision highlighted by the theoretical literature on group lending are:

1. *Adverse selection.* Poor borrowers in LDCs often cannot provide collateral to lending institutions, and even when collateral is available, legal obstacles often prevent repossessing collateral when borrowers default. Suppose that there are two types of borrowers, “safe” and “risky.” The availability of credit with only weak enforcement of repayment will tend to attract risky borrowers, whose type is unobservable to the lending institution. The resulting increase in interest rates will drive away safe borrowers, compounding the problem. Ghatak (1999) and van Tassel (1999) have argued that group lending can solve this problem by taking advantage of information villagers have of each other’s type which is unavailable to the lender.
2. *Ex ante moral hazard.* The same difficulty of enforcement and the unavailability of collateral that generate the adverse selection problem lead to an additional problem stemming from asymmetrical information. Borrowers can choose the level of effort they will exert in making

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<sup>1</sup>For general surveys of the theoretical and empirical literature, see Ghatak and Guinane (1999), Morduch (1999), and Guttman (2006).

their investment projects succeed, and can also choose the riskiness of the project that they undertake, and keep this effort level (or level of risk) unknown to the lending institution. With imperfect enforcement, borrowers have an incentive to choose a low level of effort and a risky project, since failure of the project will impose a cost primarily on the lender, not on themselves. Group lending can be used to exploit members' superior information of each other's levels of effort or riskiness of their projects, making the lender's portfolio less risky.<sup>2</sup>

3. *Ex post moral hazard* or *strategic default*. Even when a borrower's project succeeds, he or she may try to claim that the project failed and thus avoid repayment (again, due to costly enforcement and unavailability of collateral), or may simply refuse to repay. Group lending, which generally utilizes joint liability of members' debts, can solve this problem by giving incentives to members to apply social sanctions to strategic defaulters.<sup>3</sup>

This note focuses on the first approach to understanding the success of group lending, the “adverse selection” approach. A crucial theoretical result in the models of Ghatak (1999) and van Tassel (1999) is that borrowers, if allowed to form their own groups, will sort themselves into relatively homogenous groups of “safe” and “risky” borrowers, provided that they can make side-payments between themselves. The present note shows that this “positive assortative matching” result can be reversed in the presence of “dynamic incentives”—specifically, the threat of not being refinanced if the group defaults.

Section 2 demonstrates the validity of the positive assortative matching result when there is no threat of not receiving loans in the future, if the group defaults. Section 3 introduces this “refinancing threat” and shows how the positive assortative matching result is reversed under plausible conditions. Section 4 concludes.

## 2 The positive assortative matching result

Suppose there are two types of borrowers, risky (type  $a$ ) and safe (type  $b$ ). Each borrower takes a loan to finance a project which has probability of success  $p_i$  ( $i = a, b$ ) where  $p_a < p_b$ . For simplicity, it is assumed that the success or failure of one member's project is uncorrelated with the success

<sup>2</sup>See Stiglitz (1990), and Banerjee, Besley, and Guinnane (1994).

<sup>3</sup>See Besley and Coate (1995), and Armendáriz de Aghion (1999).

or failure of the other member's project. If the project succeeds, it yields income  $H$ , while if it fails, it yields zero income. The borrower takes a loan of one unit of capital, and undertakes to pay  $r > 1$  (principal plus interest) at the end of the loan time period. Two borrowers voluntarily form a group taking a group loan, in which each borrower undertakes to pay  $c$  if his or her co-member defaults. Thus the parameter  $c$  measures the degree of joint liability. The terms of the contract,  $r$  and  $c$ , are exogenous variables in this analysis.<sup>4</sup> In the event of failure of his or her own project, the borrower pays nothing (i.e., the borrower has no collateral).

Assuming that both borrowers are risk-neutral,<sup>5</sup> borrower  $i$ 's expected payoff of taking a loan together with borrower  $j$  is

$$\begin{aligned} E\pi_{ij} &= p_i p_j (H - r) + p_i (1 - p_j) (H - r - c) \\ &= p_i (H - r) - p_i (1 - p_j) c. \end{aligned} \quad (1)$$

The first term in the second line is borrower  $i$ 's expected net payoff from his own project, and the second term is his expected cost from failure of the project of borrower  $j$ . [Note that borrower  $i$  must bear (part of) the cost of borrower  $j$ 's failure,  $c$ , only if his own project succeeds. Thus the probability that he will have to pay the cost  $c$  is the probability of the event that his own project succeeds  $p_i$  and borrower  $j$ 's project fails,  $(1 - p_j)$ , i.e.,  $p_i(1 - p_j)$ .]

Thus the expected payoff to a safe borrower of taking a loan together with another safe borrower is

$$\begin{aligned} E\pi_{bb} &= p_b p_b (H - r) + p_b (1 - p_b) (H - r - c) \\ &= p_b^2 c + p_b (H - r - c). \end{aligned} \quad (2)$$

In contrast, the safe borrower's expected payoff of taking a loan with a risky borrower is

$$\begin{aligned} E\pi_{ba} &= p_b p_a (H - r) + p_b (1 - p_a) (H - r - c) \\ &= p_b p_a c + p_b (H - r - c). \end{aligned} \quad (3)$$

Subtracting (3) from (2), we have

$$E\pi_{bb} - E\pi_{ba} = p_b (p_b - p_a) c > 0. \quad (4)$$

<sup>4</sup>That is, we are not inquiring into the *optimal* contract, unlike Laffont and N'Guessan (2000) for example, but rather analyzing a generic case similar to the group lending contracts used in practice.

<sup>5</sup>I employ the standard simplifying assumption of risk-neutrality.

This expression measures the safe borrower's *relative* preference to form a group with another safe borrower. Similarly, the risky borrower's relative preference to form a group with a safe borrower is

$$E\pi_{ab} - E\pi_{aa} = p_a(p_b - p_a)c > 0 \quad (5)$$

Note that while both (4) and (5) are positive, (4) is larger than (5), since  $p_b > p_a$ . Thus, while a risky borrower would be willing to pay the r.h.s. of (5) to a safe borrower to accept him as a partner, the safe borrower would not be willing to accept such a side-payment, since she is willing to pay the r.h.s. of (4) to have another safe borrower as a partner rather than a risky borrower. In brief, *safe borrowers will be willing to pay more than risky borrowers to have safe borrowers as fellow group members*. Therefore, risky borrowers will be forced to form groups with other risky borrowers, while safe borrowers will form groups with other safe borrowers. This is the positive assortative matching result demonstrated by Ghatak (1999) and van Tassel (1999).

### 3 The effect of the refinancing threat

Let us define the concept of “group default.” A group will be said to default if it fails to repay at least  $r + c$  (which is what the group undertakes to repay in the event that only one member's project succeeds). In particular, if the group completely fails to repay (which is what would occur if both members' projects fail), it is said to default. Let us now make the following assumptions:

1. The *refinancing threat*: If the group defaults, each group member loses the opportunity to borrow in the future. (Note that we are thus introducing a long-term relationship between each group member and the lender, as well as between the group members, in contrast to the one-shot model of Section 2.)
2.  $H > c + r$ , so that a borrower whose project succeeds *can* repay his fellow member's debt if the latter defaults.

Let us compute the present value of a borrower  $i$ 's long-term relationship with the lender, to be denoted  $V_i$ . Equation (1) shows the borrower's expected payoff in each stage of this relationship. Assuming, for simplicity, an infinite time horizon, (1) now becomes

$$V_i = [p_i(H - r) - p_i(1 - p_j)c]$$

$$\begin{aligned}
& + \left\{ \frac{1 - (1 - p_i)(1 - p_j)}{r} \right\} [p_i(H - r) - p_i(1 - p_j)c] \\
& + \left\{ \frac{1 - (1 - p_i)(1 - p_j)}{r} \right\}^2 [p_i(H - r) - p_i(1 - p_j)c] \\
& + \left\{ \frac{1 - (1 - p_i)(1 - p_j)}{r} \right\}^3 [p_i(H - r) - p_i(1 - p_j)c] + \dots \\
& = \frac{p_i(H - r) - p_i(1 - p_j)c}{1 - \frac{1 - (1 - p_i)(1 - p_j)}{r}}. \tag{6}
\end{aligned}$$

The expression in the square brackets, that is repeated in each component of the above infinite series, is the same as the right-hand side of (1), the borrower's expected stage payoff. The expression in the curly brackets is the discount factor that the borrower applies to his or her expected payoff in the next stage (recall that  $r$  is the *gross* interest rate, which is one plus the net interest rate). The expression in the numerator in the curly brackets is the probability that the group does *not* default in the current stage. This is simply the probability that at least one of the two members' projects succeeds. By Assumption 1, the borrower will receive a loan in the next stage, thus providing the expected stage payoff given in the square brackets, only if the group does not default in the current stage. (If the group defaults at any stage, the borrower's payoff in all subsequent stages is zero.) Thus the appropriate discount factor to applied to a stage payoff two periods in the future is the expression in curly brackets squared, and so forth.

In the context of the refinancing threat, the positive assortative matching result, obtained by Ghatak (1999) and van Tassel (1999), requires that for  $p_i > p_j$ ,  $\partial V_i / \partial p_j > \partial V_j / \partial p_i$ . In words, the marginal benefit of having a safer partner (larger  $p_j$  for member  $i$ , and larger  $p_i$  for member  $j$ ) must be greater, the larger is the member's own probability of success. Suppose (as we assumed in Section 2) that the parameters  $H$ ,  $r$ , and  $c$  are the same for the two members, so that the two members'  $V$  functions are the same, only with the arguments  $p_i$  and  $p_j$  reversed. The above inequality then implies, for  $p_i \approx p_j$ , that

$$\frac{\partial}{\partial p_i} \frac{\partial V_i}{\partial p_j} > 0, \tag{7}$$

which, in mathematical jargon, means that the  $V$  function is supermodular.<sup>6</sup>

<sup>6</sup>Supermodularity actually would be defined in terms of the cross-partial derivative defined in the opposite order, i.e.,  $(\partial/\partial p_j)(\partial V/\partial p_i) > 0$ . Given the well-behavedness of  $V$ , however, (7) is equivalent to this condition, by Young's Theorem.

Computing the l.h.s. of (7) and simplifying, we obtain

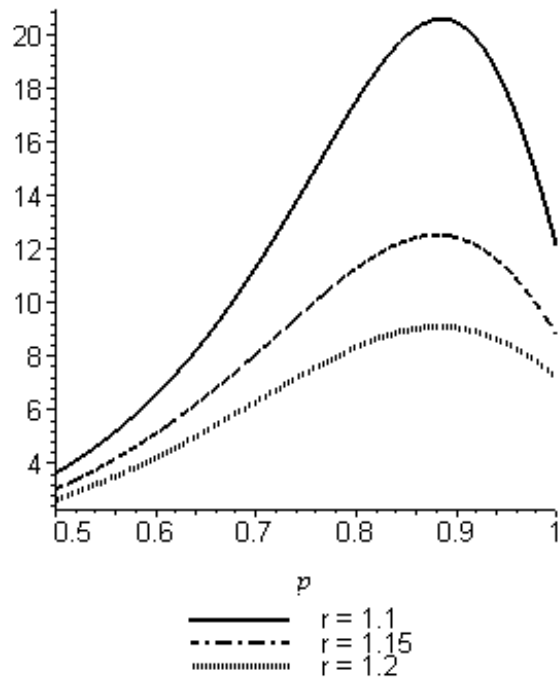
$$\frac{\partial}{\partial p_i} \frac{\partial V_i}{\partial p_j} = \frac{r[(r - p_j + p_i + p_i p_j - 2p_i r)(H - r) + (r - p_j + p_i - p_i p_j)(r - 1)c]}{(r - p_i - p_j + p_i p_j)^3},$$

whose sign is ambiguous. This ambiguity in the sign of the l.h.s. of (7) means that the  $V$  function is not necessarily supermodular, implying that positive assortative matching need not occur.

If the net interest rate is zero and therefore  $r = 1$ , the above expression simplifies to an unambiguously positive value. But for higher interest rates, and particularly for relatively large values of  $p_i$  and of  $H$ , this expression can be negative. To illustrate, Figure 1 shows  $\partial V_i / \partial p_j$ , which is member  $i$ 's marginal benefit of having a safer partner, as a function of  $p_i$ , for three alternative interest rates,  $r = 1.1, 1.15$ , and  $1.2$  (corresponding to net interest rates of 10, 15, and 20 percent, which are in the range of interest rates commonly encountered in microfinance programs). The other assumed parameter values are  $H = 2.5$ ,  $c = r$  (implying full joint liability, since each member undertakes to pay  $r$  if his or her project succeeds), and  $p_j = p_i$ . (The shape of these functions is very similar if we use a fixed value for  $p_j$ , but this would contradict the assumption that  $p_i$  and  $p_j$  are almost equal, enabling us to use the differential approach employed here.)

As the figure shows, when the two members' probabilities of success approach 0.9, member  $i$ 's marginal benefit of having a safer partner  $j$  declines as a function of member  $i$ 's own probability of success. For larger values of  $H$ , the peak in these functions is reached at a lower  $p_i$ . When  $\partial V_i / \partial p_j$  declines as a function of  $p_i$ , the l.h.s. of (7) is negative, and the  $V$  function is submodular, not supermodular. In this region, *negative* assortative matching would occur: groups would be mixed in terms of the riskiness of their members.

In practice, the two members' probabilities of success are generally quite different, invalidating the use of partial and cross-partial derivatives. If we simply evaluate  $V_i$  and  $V_j$  for assumed values of  $p_i$  and  $p_j$ , we can assess the likelihood of positive assortative matching in such cases. Consider a numerical example. Using the same assumed value for  $H$  as we assumed above, and assuming  $r = c = 1.15$ ,  $p_i = 0.95$  and  $p_j = 0.7$ , Table 1 shows the values of  $V_i$  and  $V_j$  and the derived side-payments that each member would be willing to pay to have a safe partner (whose probability of success is 0.95 rather than 0.7). The left-hand entry in the first two columns is the present value of the payoffs  $V$  of the "row player," and the right-hand entry is the  $V$  for the "column player," as is customary in payoff matrices in game theory. As the table shows, the more risky partner is willing to pay

Figure 1:  $\partial V_i / \partial p_j$  as function of  $p_i$

more to have a safe partner, 2.94, than the safe partner is willing to pay, 2.61. Therefore *negative* assortative matching would occur: groups would be mixed, rather than homogenous.

Table 1  
Present Values of Payoffs  $V$  of Safe and Risky Partners

	Safe	Risky	Marginal benefit of safe partner
Safe	9.26, 9.26	6.65, 6.31	2.61
Risky	6.31, 6.65	3.37, 3.37	2.94

#### 4 Concluding remarks

We have found that, if the members of the group are subject to a refinancing threat, the positive assortative matching result of Ghatak (1999) and van Tassel (1999) does not necessarily hold. The intuition underlying the positive assortative matching result and its reversal with dynamic incentives can be explained as follows. Consider first the one-shot case, without dynamic incentives. In this case, a safe borrower will value having another safe borrower as a fellow group member more than a risky borrower will value having a safe borrower as a peer, since a risky borrower (by definition) has a larger probability of defaulting and thus a lower probability of having to pay back the debts incurred by his peer if the peer defaults. Thus the peer's type (safe or risky) matters *less* to a risky borrower than it does to a safe borrower.

Now consider the effect of the refinancing threat. The probability that the group will not receive loans in the future equals the *product* of the failure probabilities of the two members' projects (since the group defaults only if *both* members' projects fail).<sup>7</sup> This probability is  $(1 - p_b)^2$  when both members are safe borrowers, and  $(1 - p_a)(1 - p_b)$  when one member is a safe borrower and the other is a risky borrower. Thus the marginal effect on the probability of group default, to a safe borrower, of having a risky rather than a safe peer, is

$$(1 - p_a)(1 - p_b) - (1 - p_b)^2 = (p_b - p_a)(1 - p_b). \quad (8)$$

<sup>7</sup>This assumes, of course, that if member 1's project succeeds and member 2's project fails, then member 1 will repay his committed share of member 2's debt,  $c$ , thus avoiding group default. A sufficient, but not quite necessary condition, for repaying  $c$  to be optimal to member 1 is  $V > c$ .

The corresponding marginal effect on the probability of group default, to a risky borrower, of having a risky rather than a safe peer is

$$(1 - p_a)^2 - (1 - p_a)(1 - p_b) = (p_b - p_a)(1 - p_a). \quad (9)$$

Since  $p_b > p_a$ , (8) is less than (9). Thus the marginal effect on the probability of group default, to a safe borrower is less than the corresponding marginal effect, to a risky borrower. This causes the risky borrower to be willing to pay *more* to have a safe borrower as a peer, than a safe borrower will be willing to pay. This, in turn, leads to *negative* assortative matching.

It should be noted, however, that if side-payments are not feasible, we would always expect positive assortative matching. Both safe and risky borrowers prefer safe borrowers as peers. Therefore, without side-payments, safe borrowers would choose to form groups together, leaving the risky borrowers to form groups among themselves. Thus, contrary to the arguments of Ghatak (1999) and van Tassel (1999), positive assortative matching is more certain when side-payments between borrowers are *not* feasible.

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