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Systemic Risk-Taking: Amplification Effects, Externalities, and Regulatory Responses

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Abstract: This paper develops a simple macroeconomic model of systemic risk in the form of financial accelerator effects: adverse developments in financial markets and in the real economy mutually reinforce each other and lead to a feedback cycle of falling asset prices, deteriorating balance sheets and tightening financing conditions. We show that decentralized agents choose to expose themselves to financial accelerator effects to a socially inefficient extent and do not take on sufficient insurance against systemic risk even if given access to a complete ex-ante insurance market. We use the framework to shed light on a number of current policy issues: First, we develop a new analytical framework of macro-prudential capital adequacy requirements that take into account systemic risk by employing an externality pricing kernel. Second, we show that agents employ ex-ante risk markets to fully undo any expected government bailout. Finally, we find that constrained market participants face socially insufficient incentives to raise more capital during systemic crises.

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1 Introduction

This paper develops a simple model of systemic risk in the form of financial amplification effects that arise in response to strong adverse shocks. Risk-neutral bankers raise finance from households and invest in risky projects. When the aggregate return on the projects is low, their liquid net worth is insufficient to meet the contracted repayments and financial constraints force them to engage in fire sales, i.e. they sell some of the projects at a price that is below their marginal product. This triggers financial amplification effects: the more bankers sell, the larger the decline in asset prices and the lower the amount of liquidity they raise from a given amount of sales, requiring in turn further sales to meet a given repayment obligation (see figure 1). We characterize the resulting downward spiral in the economy as systemic risk.[1]

Ex ante, the privately optimal financial contract for bankers trades off the efficiency cost of fire sales against the premium demanded by households for taking on aggregate risk. Individual bankers do not internalize their contribution to aggregate price declines and therefore impose pecuniary externalities on each other when they engage in fire sales. This induces them to take on socially excessive systemic risk in their ex ante financing decisions.[2]

We employ our model to shed light on a number of policy issues that have been debated in the aftermath of the recent financial crisis:

First, we develop a theoretical framework of macro-prudential regulation that induces individual bankers to internalize their contribution to systemic risk. We characterize an externality pricing kernel that captures the state-contingent magnitude of systemic externalities and that can be used to price the externalities imposed by financial claims or real investment opportunities. In states when financial constraints are loose, the externality kernel is zero; in constrained states the externality kernel captures the social cost of amplification effects created by a unit payoff. A policymaker who charges a Pigovian tax to offset the externalities or imposes equivalent regulatory measures on bankers can restore constrained Pareto efficiency in the economy.

[1]This is in accordance with the description of systemic risk by the Bank for International Settlements: exogenous shocks to financial institutions that have common risk exposure are endogenously amplified because of wide-spread financial distress (see e.g. Borio, 2003, for a discussion).

[2]There is a clear analogy to more traditional forms of externalities: for example, an entrepreneur who creates pollution derives private benefits from his activities, but society at large bears some of the costs. In our example, a banker who exposes himself to the risk of fire-sales obtains higher profits in good states of nature, but the economy at large suffers from the amplification effects that are triggered by fire-sales in bad states of nature. If he limited his risk-taking, he would bear all the costs in terms of foregone profits, but the economy at large would benefit from the mitigation of fire sales and from greater financial stability.

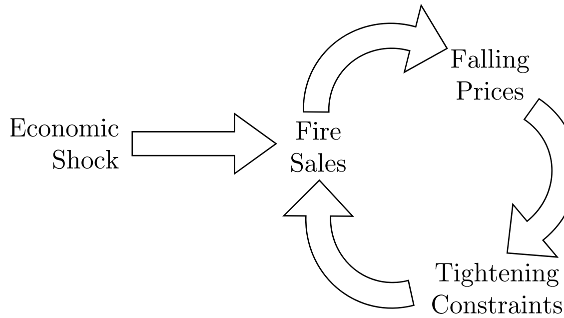

Figure 1: Financial amplification under binding financial constraints

Second, we derive a bailout neutrality result: we show that unregulated bankers will employ ex-ante risk markets to fully undo any expected lump-sum government transfer that aims to mitigate financial amplification effects. Undoing such transfers is optimal for bankers since the equilibrium with excessive systemic risk constitutes their private optimum.

Third, we find that individual bankers undervalue the social benefits of raising capital during systemic crises because they do not internalize the positive effects of reducing their fire sales on the rest of the banking system. This provides a policy rationale for mandatory capital injections.

Our paper also illustrates the conceptual difference between systematic risk and systemic risk: Bankers in our model are always subject to systematic risk (i.e. to aggregate, undiversifiable market risk). However, systemic risk only arises when the banking sector as a whole experiences binding financial constraints and financial amplification effects are triggered.

The setting in which we describe our results is an economy with three time periods $t = 0, 1, 2$ and two types of agents, bankers and households. The economy experiences an aggregate shock that is realized at the beginning of period 1. Bankers are risk-neutral and raise finance in a complete market of Arrow securities in period 0 for a lumpy investment project that yields a risky payoff in period 1 and a safe payoff in period 2. As in Kiyotaki and Moore (1997), we assume that financial promises need to be secured and that bankers can use the asset value of the project as collateral, but not the contemporary return.[3] This implies that bankers do have collateral to back up the Arrow securities due in period 1, but they cannot commit to repayments in period 2 since the asset value of all projects is zero in the final period – no borrowing between periods 1 and 2 can be sustained. However, bankers can raise finance in period 1 by selling a fraction of their projects at the prevailing market price to the household sector. In period 2, bankers consume the payoff on their remaining

[3]Kiyotaki and Moore (1997), building on Hart and Moore (1994), motivate this by observing that the owners of a project could threaten to withdraw their labor and thereby destroy the contemporary return.

asset holdings and perish.

Households come in two generations. First-generation households live from period 0 to 1 and provide finance to bankers in the market for Arrow securities. They are risk-averse so their demand for securities contingent on a particular state of nature is downward-sloping. This makes it costly for bankers to share risk with them. Second-generation households are risk-neutral, live from period 1 to 2 and have access to a technology that employs the assets of bankers but that is less productive and subject to decreasing returns-to-scale. Therefore the asset demand of second-generation households is downward-sloping, and it is costly for bankers to sell assets.

If the initial investment requirement of bankers is sufficiently small, they promise a fixed payment to first-generation households, which they finance from their period 1 payoff. They absorb all aggregate risk and do not engage in fire sales in period 1. In this case the decentralized equilibrium in the economy is socially efficient.

For a larger initial investment requirement, the period 1 payoff of bankers in low states of nature does not allow them to make a fixed payment to first-generation households without engaging in costly asset sales. Bankers therefore need to find the optimal trade-off between costly risk sharing with first-generation households and asset sales at a price below their marginal product to second-generation households.

Our main result is that bankers in the decentralized equilibrium of the described economy insure too little in ex ante risk markets and engage in excessive fire sales in ex post asset markets once an adverse shock has materialized. The reason for this distortion is that atomistic bankers take prices in the economy as given and do not internalize the pecuniary externalities that their fire sales give rise to. Under complete markets pecuniary externalities do not have efficiency implications because the relative marginal valuations of all goods among all agents in the economy are equated so that a redistribution cannot achieve a Pareto improvement. In the described setting, by contrast, binding financial constraints imply that bankers value productive assets more highly than households. A constrained social planner internalizes that reducing fire sales keeps asset prices more elevated, which mitigates the financial constraints on bankers. (By contrast, atomistic bankers just take asset prices as given.) Therefore the planner engages in more ex ante insurance and fewer fire sales than decentralized agents.

Our inefficiency result relies crucially on the assumption that bankers cannot borrow against the payoffs of their asset holdings in the final period. Otherwise bankers would be the natural holders of all assets, since they have the most productive technology to operate them, fire sales would not occur, and the equilibrium would be efficient. Furthermore, our finding relies on the assumption that second-generation households are not alive in period

0 and cannot provide risk-neutral insurance in that period. Otherwise bankers in period 0 would fully insure against having to engage in fire sales in period 1 and would achieve a Pareto efficient allocation.

Literature Our work builds on the literature on financial amplification and fire sales as described by Fisher (1933), Bernanke and Gertler (1990), Shleifer and Vishny (1997) or Kiyotaki and Moore (1997). Specifically, our model is a simplified version of Kiyotaki and Moore (1997). In this literature, it is common to assume that financially constrained bankers/entrepreneurs only have access to uncontingent forms of finance. If they had access to complete and risk-neutral insurance markets, bankers/entrepreneurs would fully insure against the risk of becoming constrained and no financial amplification effects would occur in case of adverse shocks (Krishnamurthy, 2003). This paper shows that risk aversion among the providers of finance is sufficient to break this result, as bankers trade off the costs of binding financial constraints and of purchasing insurance and choose an interior optimum.

The paper also builds on the literature on the generic inefficiency of the decentralized equilibrium under incomplete markets (Hart, 1975; Stiglitz, 1982; Geanakoplos and Polemarchakis, 1986), which includes more recent seminal contributions by Gromb and Vayanos (2002), Caballero and Krishnamurthy (2003) and Lorenzoni (2008). Gromb and Vayanos (2002) analyze financially constrained arbitrageurs and show that they generally fail to engage in the socially efficient amount of arbitrage between two risky assets because they do not internalize the pecuniary externalities involved in fire sales when financial constraints are binding. Aside from the two risky assets, arbitrageurs in their model only have access to uncontingent bonds.

In Caballero and Krishnamurthy (2003) and Lorenzoni (2008), entrepreneurs raise finance for a risky investment project in a risk-neutral security market and face the risk of binding financial constraints in a subsequent period. Caballero and Krishnamurthy (2003) investigate the financing and investment decisions in a small open emerging economy in which binding future constraints result in exchange rate depreciations. Lorenzoni (2008) focuses on the aggregate level of investment in a simplified Kiyotaki-Moore economy similar to ours, in which binding constraints lead to fire sales and asset price declines. In both works, entrepreneurs engage in excessive investment because of pecuniary externalities that arise from future binding constraints.

However, their results rely on the assumption that binding financial constraints in the initial security market prevent optimal insurance against future shocks: in Caballero and Krishnamurthy (2003), entrepreneurs would like to commit to higher repayments in good states of nature, but limited collateral prevents them from doing so; in Lorenzoni (2008), entrepreneurs would like to purchase more insurance against low states of nature, but limited commitment prevents the sellers of insurance from providing it.[4]

In our paper, by contrast, bankers have access to a complete and unconstrained Arrow security market in the inital period, which they can use to share risk with risk-averse households. The explicit focus on the unconstrained trade-off between risk and return makes our framework well suited for studying optimal risk-taking and price-based macroprudential regulation of systemic externalities. Furthermore, we present a number of additional new results, including on the ex-ante effects of bailout transfers and on the incentives for constrained bankers to raise new capital.

A number of recent empirical papers document the importance of financial amplification effects. For example, Adrian and Brunnermeier (2008) show that VaR – a measure for the riskiness of a financial institution's assets – rises strongly when another institution is in distress. They also document that financial institutions that increase their exposure to systemic risk raise their expected return, consistent with our theoretical model. Adrian and Shin (2010) find that leverage among investment banks is strongly pro-cyclical, implying that they take on more risk in good times and sell off risky assets in bad times. Benmelech and Bergman (2011) provide evidence for fire-sale externalities in the airline sector.

The rest of the paper is structured as follows. The following section describes our model setup. Section 3 analyzes the decentralized equilibrium of the economy and the dynamics of financial amplification when financing constraints are binding. Section 4 analyzes the social efficiency of the decentralized equilibrium and presents a new framework of macroprudential regulation. In section 5 we study extensions of our baseline model to develop our results on bailout neutrality and on the incentives for raising capital. Section 6 concludes. The appendix contains a detailed discussion of some of the technical assumptions and proofs of our model.

2 Model

Our model economy consists of three time periods $t = 0, 1, 2$ and is inhabited by two categories of atomistic agents, bankers and households. Banking entrepreneurs represent the consolidated productive sector of the economy and could alternatively be interpreted as entrepreneurs – the important characteristic is that they make financing decisions and are subject to business risk and financial constraints. We will call them in short "bankers."

[4]One important implication of this setup is that sufficient provision of public liquidity would address the imperfections in the security market and would restore constrained social efficiency as in Holmström and Tirole (1998). This would not help in our setup.

Households come in two generations; they are less productive than bankers, but they receive endowments and therefore have the ability to provide finance to bankers. There are two types of goods, a homogeneous consumption good and a productive asset. In period 1, a random state of nature $\omega \in \Omega$ is realized, where Ω is a set of all possible outcomes. The productivity of bankers' assets in period 1 is given by a random variable A_1^ω, which is continuously distributed over the interval $[A^{\min}, A^{\max}] \subseteq \mathbb{R}^+$ with density function $g(A)$, and which satisfies the normalization $E[A_1^\omega] = \bar{A}_1 = 1$.

Bankers Bankers are risk-neutral and value consumption in periods 1 and 2 according to the function

$$V = E[c_{1,b}^\omega + c_{2,b}^\omega] \tag{1}$$

where we use the subindex 'b' for banker-specific consumption variables. In period 0, they have access to a lumpy investment technology that allows them to invest αt_1 consumption goods and obtain t_1 units of productive capital goods. We can think of this as planting a seed that costs α on t_1 units of land each. (We will discuss a generalization of this later.) They have no endowment, so they need to finance their period 0 investment by selling financial claims in a complete one-period market of Arrow securities contingent on the state of nature $\omega \in \Omega$. We denote the amount to be repaid in state ω of period 1 as b_1^ω and the stochastic discount factor (or pricing kernel) at which the claims are priced in period 0 as m_1^ω.The resulting period 0 budget constraint is

$$\alpha t_1 = E[m_1^\omega b_1^\omega] \tag{2}$$

In period 1, each unit of the capital good produces a stochastic net dividend A_1^ω, which depends on the state of nature ω,

Bankers are subject to a commitment problem that limits what they can pledge to repay. Specifically, we follow Kiyotaki and Moore (1997) in assuming that when they enter financial contracts, they can only pledge the market value but not the dividend income of their asset holdings next period.[5] Since the economy ends after period 2, the asset price ex dividend is zero in that period and bankers have no collateral to pledge in period 1, i.e. no borrowing between periods 1 and 2 can be sustained. We therefore set w.l.o.g. $b_2^\omega = 0$. Following the same argument, bankers do have collateral to offer between periods 0 and 1, which they use to back up their promises b_1^ω.

In period 1, bankers cannot borrow, but they have access to a market in which they can trade their productive assets at price q_1^ω. As we will see below, the asset sales of bankers in

[5]Kiyotaki and Moore (1997) motivate this by the notion that bankers could threaten to withdraw their labor in the period in which lenders try to seize the asset, which would destroy all contemporaneous output.

this market share certain characteristics of fire-sales; therefore we denote the quantity that bankers sell as fire-sales f_1^ω.

We make two simplifying assumptions, which are formalized in appendix A.1: First we assume that the collateral of bankers in period 1 is always sufficient to back up their optimal period 0 financial promises b_1^ω. Second, we assume that the initial investment requirement αt_1 is sufficiently low that bankers' optimal fire sales satisfy $f_1^\omega \leq t_1 \forall \omega$. We can therefore omit the two constraints on period 0 borrowing and on fire sales from the maximization problem below. However, note that neither assumption is critical to obtain the basic results of our paper.

Accounting for the promised repayment b_1^ω on the Arrow securities that they issued, the period 1 budget constraint of bankers is

$$c_{1,b}^\omega + b_1^\omega = A_1^\omega t_1 + q_1^\omega f_1^\omega \tag{3}$$

Given their linear preferences, bankers would like to substitute consumption between periods 1 and 2 at a rate of unity. We impose a non-negativity constraint on period 1 consumption $c_{1,b}^\omega \geq 0$ to prevent them from using this device to circumvent the borrowing constraint that they face.

In period 2, bankers employ their remaining asset holdings $(t_1 - f_1^\omega)$ in production, and they consume the resulting output $c_{2,b}^\omega = \bar{A}_2(t_1 - f_1^\omega)$, where $\bar{A}_2 > \bar{A}_1$ since period 2 reflects the entire future of the economy. The resulting optimization problem for bankers is

$$\max_{\{b_1^\omega, c_{1,b}^\omega, f_1^\omega\}} E\left[c_{1,b}^\omega + \bar{A}_2(t_1 - f_1^\omega)\right] \quad \text{s.t. (2), (3) and } c_{1,b}^\omega \geq 0 \tag{4}$$

First-Generation Households We assume that there are two generations of households that live for two periods each. The first generation lives across periods 0 and 1. They are risk averse and derive utility from consumption according to the function

$$U^\omega = u(c_{0,h}) + E[u(c_{1,h}^\omega)]$$

where $u(\cdot)$ is a standard neo-classical utility function. We use the sub-index 'h' for first-generation households. They receive an endowment e every period that satisfies $e > \alpha t_1$. In period 0 they buy a bundle $\{b_{1,h}^\omega\}$ of Arrow securities that offer a contingent repayment $b_{1,h}^\omega$ in period 1. Given the stochastic discount factor $\{m_1^\omega\}$ at which Arrow securities are priced in the market, the total outlay of first generation households in period 0 is $E[m_1^\omega b_{1,h}^\omega]$.

We denote their optimization problem as

$$\max_{\{b_{1,h}^\omega\}} u\left(e - E[m_1^\omega b_{1,h}^\omega]\right) + E\left[u(e + b_{1,h}^\omega)\right] \tag{5}$$

The Euler equation that captures their demand for Arrow securities contingent on state ω is

$$FOC(b_1^\omega): \; m_1^\omega = \frac{u'(c_{1,h}^\omega)}{u'(c_{0,h})} \tag{6}$$

Demand for Arrow securities is downward-sloping, implying that $dm_1^\omega / db_{1,h}^\omega < 0$. Furthermore, we assume that the functional form of $u(\cdot)$ and the parameters of the model are such that $dm_1^\omega b_{1,h}^\omega / db_{1,h}^\omega > 0$. The technical condition for this is listed as assumption A.2 in the appendix.

Remark: First generation households could alternatively be interpreted as entrepreneurs who are unconstrained and who have a competing use for funds in a concave production technology that mirrors the concave utility function of households.

Second-Generation Households Second generation households live from period 1 to period 2. They value consumption according to the linear utility function

$$W = E\left[c_{1,l}^\omega + c_{2,l}^\omega\right]$$

where the sub-index 'l' denotes variables of second-generation households. They receive an endowment e every period and buy $f_{1,l}^\omega$ productive assets at the given market price q_1^ω in period 1. As in Lorenzoni (2008), they employ their asset holdings in period 2 production using a decreasing returns-to-scale production function $F(\cdot)$ that satisfies $F'(0) = \bar{A}_2$ and $F'' < 0$, i.e. their marginal productivity is equal to the productivity of bankers at zero, but declines in the amount of assets purchased – households are less productive than bankers for any positive amount of assets employed.

The resulting optimization problem for second generation households is

$$\max_{\{f_{1,l}^\omega\}} E\left[\left(e - q_1^\omega f_{1,l}^\omega\right) + \left(e + F(f_{1,l}^\omega)\right)\right] \tag{7}$$

The first-order condition yields the demand for productive assets

$$q_1^\omega = F'(f_{1,l}^\omega) \tag{8}$$

Their demand function is downward-sloping, implying that $dq_1^\omega / df_{1,l}^\omega < 0$. Furthermore, we assume the functional form of $F(\cdot)$ and the parameters of the model are such that the amount spent on asset purchases is strictly increasing in $f_{1,l}^\omega$, i.e. $dq_1^\omega f_{1,l}^\omega / df_{1,l}^\omega > 0$. The technical condition for this is described in assumption A.3 in the appendix.

The strictly monotonic relationship between the quantity f of assets purchased and the amount spent on such purchases $s = f \cdot q = f \cdot F'(f)$ allows us to define a function $f(s)$

that expresses the amount of asset purchases resulting from an amount $s \geq 0$ spent on such purchases as the implicit solution to

$$s = f(s) \cdot F'(f(s)) \tag{9}$$

For later use, we define $f(s) = 0$ for $s < 0$. We denote the corresponding asset price function $q(s) = F'(f(s))$. If bankers fire-sell all their productive assets, the asset price declines to $q^{\min} = F'(t_1)$ and they could raise a maximum amount

$$s^{\max} = t_1 q^{\min} = t_1 \cdot F'(t_1)$$

The functions $f(s)$ and $q(s)$ are therefore both defined over the interval $(-\infty, s^{\max}]$. For non-positive values of $s \leq 0$ they return $f(s) = 0$ and $q(s) = \bar{A}_2$. For positive values $s \in (0, s^{\max}]$, $f(s)$ increases in a convex manner from 0 to t_1 and $q(s)$ decreases from $\bar{A}_2$ to $q^{\min}$.

The magnitude $s^{\max}$ also represents what first-generation households could obtain if they seize all t_1 assets from bankers in period 1 and re-sell them to second-generation households.

Remark: In the described setup, the demand of second-generation households for productive assets is downward-sloping because their production technology exhibits decreasing returns to scale. We could obtain similar results if they had concave utility $u(\cdot)$ and a linear production technology $\bar{A}_2 f$. In that case asset demand would be defined by the optimality condition $q = \bar{A}_2 \cdot \frac{u'(e+\bar{A}_2 f)}{u'(e-qf)}$ and would be downward-sloping because households dislike an unsmooth consumption profile.

3 Decentralized Equilibrium

An equilibrium in the economy consists of a set of allocations $\left(c_{0,h}, c^{\omega}_{1,h}, c^{\omega}_{1,b}, c^{\omega}_{2,b}, c^{\omega}_{1,l}, c^{\omega}_{2,l}, b^{\omega}_1, b^{\omega}_{1,h}, f^{\omega}_1, f^{\omega}_{1,l}\right)$ and prices $(m^{\omega}_1, q^{\omega}_1)$ which satisfy the maximization problems (4), (5), (7) of all three agents as well as the market-clearing conditions for Arrow securities $b^{\omega}_1 = b^{\omega}_{1,h}$ and the asset market $f^{\omega}_1 = f^{\omega}_{1,l}\ \forall\ \omega$.

3.1 Backward Induction: Period 1 Equilibrium

We solve the problem of bankers by backward induction: we first analyze their optimal period 1 and 2 allocations, given that the state of the world ω is realized at the beginning of period 1; then we proceed to solve for the optimal financing decision in period 0.

After the productivity shock ω has been realized, denote by $V(a^{\omega})$ the utility that a banker obtains from his net liquid asset holdings $a^{\omega} = A^{\omega}_1 t_1 - b^{\omega}_1$ in the beginning of period

1. We denote the Lagrangian of the associated optimization problem as follows. (Since there are no further shocks after period 1, we drop the superscript ω for ease of notation.)

$$V(a) = \max_{\{c_{1,b}, f_1\}} c_{1,b} + \bar{A}_2(t_1 - f_1) - \mu \left[c_{1,b} - a - q_1 f_1\right] + \lambda c_{1,b} \tag{10}$$

The first order conditions are

$$\begin{aligned} \text{FOC}(c_{1,b}): \quad & \mu = 1 + \lambda \\ \text{FOC}(f_1): \quad & \bar{A}_2 = \mu q_1 \end{aligned}$$

Depending on the amount of initial liquid assets a at the beginning of period 1, we distinguish two equilibria:

Unconstrained equilibrium for $a \geq 0$: For non-negative liquid asset holdings at the beginning of period 1, the optimum allocation of bankers is unconstrained: they consume their liquid wealth in period 1 $c_{1,b} = a$ and do not engage in fire sales $f_1 = 0$. In period 2, they consume their production $c_{2,b} = \bar{A}_2 t_1$. The shadow prices satisfy $\mu = 1$ and $\lambda = 0$. The allocation $f_1 = 0$ together with a price $q_1 = \bar{A}_2$ also constitutes an optimum for second generation households.

Constrained equilibrium for $-s^{\max} \leq a < 0$: For negative liquid asset holdings, i.e. when the output of bankers in period 1 is insufficient to cover their payment obligation b_1, bankers would like to roll over debt into period 2 but are prevented from doing so by the binding borrowing constraint. We denote the liquidity shortfall $s = -a$. Bankers choose period 1 consumption $c_{1,b} = 0$ and engage in asset sales of $f(s)$ at price $q(s)$ so as to cover the liquidity shortfall $s = q(s)f(s)$.

In period 2, they consume the output from their remaining asset holdings $c_{2,b} = \bar{A}_2(t_1 - f_1)$. Second-generation households are willing to buy a level $f(s) > 0$ of assets if the price declines sufficiently below $\bar{A}_2$ so as meet their optimality condition $q_1 = F'(f(s))$. Since bankers sell assets at prices that are below their marginal product, we call these sales "fire sales." The shadow price of liquidity of bankers is $\mu = \bar{A}_2/q > 1$, reflecting that an additional unit of liquidity could buy up $1/q$ assets and earn a return $\bar{A}_2$.

Comparative Statics and Amplification Effects

Figure 2 depicts a comparative static analysis of the economy's equilibrium in period 1 for a fixed repayment obligation $\bar{b}_1$. The lower productivity A_1^ω, the lower the liquidity of bankers (left panel). If bankers produce less than the debt level $A_1^\omega t_1 < \bar{b}_1$, they experience binding constraints. As a result, they have to engage in fire sales of some of their productive asset holdings (center panel), which reduces the equilibrium price q_1 (right panel).

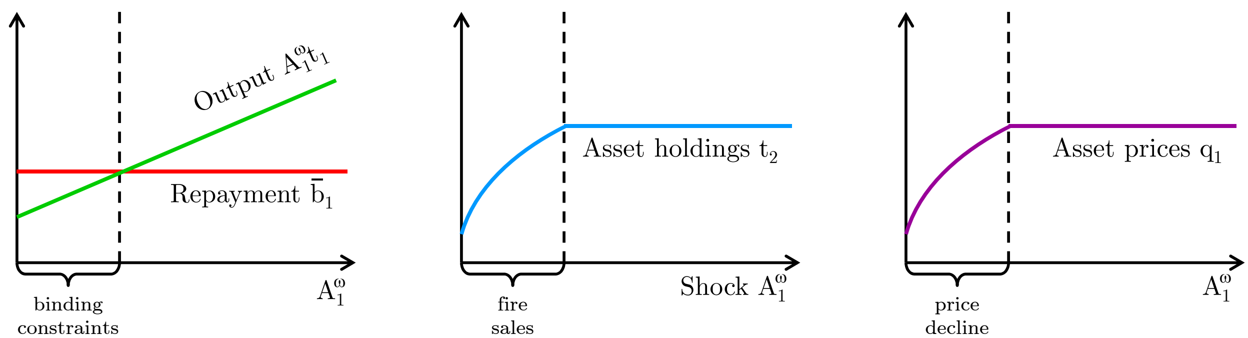

Figure 2: Fire sales and price declines as a function of A_1^ω for $b_1^\omega = \bar{b}_1$

The effects of shocks under this constrained regime are magnified by financial amplification: suppose that bankers are constrained, selling f_1 to meet their period 1 repayment obligation, and suddenly experience a small shock $ds > 0$ to their liquidity position. The partial equilibrium effect is that they are forced to fire-sell an additional $\frac{ds}{q_1}$ of their productive assets. This sale depresses the price q_1 by $\frac{ds}{q_1} \cdot \frac{\partial q_1}{\partial f_1} < 0$. By implication bankers receive $\frac{ds}{q_1} \cdot \frac{\partial q_1}{\partial f_1} \cdot f_1 < 0$ less on their prior fire sales f_1 and need to increase sales by $\frac{ds}{q_1} \cdot \left(\frac{\partial q_1}{\partial f_1} \cdot \frac{f_1}{q_1}\right) = \frac{ds}{q_1} \cdot \eta_{qf}$, leading to further price declines $\frac{ds}{q_1} \cdot \eta_{qf} \cdot \frac{\partial q_1}{\partial f_1}$, a further reduction in revenues from asset sales, further fire sales $\frac{ds}{q_1} \cdot \eta_{qf} \cdot \left(\frac{\partial q_1}{\partial f_1} \cdot \frac{f_1}{q_1}\right) = \frac{ds}{q_1} \cdot (\eta_{qf})^2$ and so on. In general equilibrium, the total effect of the shock is to lead to asset sales of

$$\frac{df_1}{ds} = \frac{1}{q_1} \cdot \frac{1}{1 - \eta_{qf}} \tag{11}$$

Note that this expression can also be obtained by implicitly differentiating equation (9). The second factor in the expression is (by assumption A.3) always greater than 1 and captures the effects of financial amplification.

3.2 Period 0 Financing Decisions

First-generation households consume $c_{0,h} = e - \alpha t_1$ in period 0 and $c_{1,h}^\omega = e + b_1^\omega$ in state ω of period 1. Following optimality condition (6), their pricing kernel m_1^ω is a function of the payment b_1^ω they receive in state ω of period 1,

$$m_1^\omega = m_1(b_1^\omega) = \frac{u'(e + b_1^\omega)}{u'(e - \alpha t_1)} \tag{12}$$

The period 0 optimization problem of bankers can be reformulated by employing the definition of V in equation (10),

$$\max_{\{b_1^\omega\}} E\left\{V\left(A_1^\omega t_1 - b_1^\omega\right)\right\} \quad \text{s.t.} \quad \alpha t_1 = E[m_1^\omega b_1^\omega] \tag{13}$$

Assigning a shadow price of ν to the period 0 budget constraint, the first-order condition of the Lagrangian to this problem for security issuance in a given state ω is

$$V'(a^{\omega}) = \nu m_1^{\omega} \tag{14}$$

Observe that $V'(a^{\omega}) = \mu^{\omega} = \frac{\bar{A}_2}{q(-a)}$ reflects the shadow price of liquidity of bankers in state ω of period 1, and ν reflects the shadow cost of raising funds in period 0.

Substituting for m_1^{ω} according to (12), we define an optimal repayment function $b_1(\nu; A_1)$ for every $A_1 \in [A^{\min}, A^{\max}]$ as the solution to the implicit equation

$$\frac{\bar{A}_2}{q(b_1 - A_1 t_1)} = \nu \cdot \frac{u'(e + b_1)}{u'(e - \alpha t_1)} \tag{15}$$

If the shadow cost ν of raising funds in period 0 is below a threshold $\hat{\nu}(A_1) = \frac{u'(e-\alpha t_1)}{u'(e+A_1 t_1)}$, then there are no fire-sales, so $q(\cdot) = \bar{A}_2$ and the left-hand side of the equation equals 1. The optimal repayment function for $\nu \in (0, \hat{\nu}(A_1)]$ can be explicitly characterized,

$$b_1(\nu; A_1) = b_1^{unc}(\nu) = u'^{-1}\left[\frac{u'(e - \alpha t_1)}{\nu}\right] - e \tag{16}$$

Note that over the interval $(0, \hat{\nu}(A_1)]$, the right-hand side of this function is independent of A_1 and is strictly increasing in ν. It satisfies $\lim_{\nu \to 0} b_1^{unc}(\nu) = -e$ and $b_1^{unc}(\hat{\nu}(A_1)) = A_1 t_1$.

For ν exceeding the threshold $\hat{\nu}(A_1)$, bankers find it optimal to promise a period 1 repayment $b_1 > A_1 t_1$ that exceeds their asset income, which implies that they must raise some liquidity through fire-sales to repay first-generation households. The left-hand side of equation (15) is then strictly increasing in b_1; for $b_1 = A_1 t_1$ it equals 1, and for $b_1 = A_1 t_1 + s^{\max}$ it is $\bar{A}_2 / q^{\min}$.

For a given ν, the right-hand side is strictly decreasing in b_1 from ∞ for $b_1 \to -e$ to 0 for $b_1 \to \infty$ because of the Inada conditions on the utility function of first-generation households. As long as $\nu \in (\hat{\nu}(A_1), \bar{\nu}(A_1)]$, the equation has a unique solution that satisfies $b_1(\nu; A_1) \in (A_1 t_1, A_1 t_1 + s^{\max}]$ and that is strictly increasing in both ν and A_1. The threshold value $\nu^{\max}(A_1) = \bar{A}_2 / q^{\min} \cdot \frac{u'(e-\alpha t_1)}{u'(e+A_1 t_1 + s^{\max})}$ is the maximum shadow price of period 0 liquidity for which a solution to equation (15) is defined for a given A_1 – a higher value of ν would induce bankers to attempt to raise more liquidity than $s^{\max}$ through fire sales, i.e. more than they can raise given the quantity t_1 of assets that they are holding.

For simplicity we limit our analysis to values of

$$\nu \leq \nu^{\max} = \min\left\{\bar{\nu}(A^{\min}), \hat{\nu}(A^{\max})\right\} \tag{17}$$

The condition $\nu \leq \bar{\nu}(A^{\min})$ guarantees that bankers will not attempt to fire sell more assets than they own in the lowest state of nature, and by implication in all other states of nature.

The condition $\nu \leq \hat{\nu}(A^{\max})$ implies that bankers will be unconstrained at least in the highest state of nature, and by implication that there are some states of nature in which they will not engage in fire-sales.[6]

In summary, the equilibrium condition (15) defines a continuous function $b_1 : (0, \nu^{\max}] \times [A^{\min}, A^{\max}] \to (-e, A^{\min}t_1 + s^{max}]$ that satisfies $\partial b_1/\partial \nu > 0$ and $\partial b_1/\partial A_1 \geq 0$. (Bankers promise to repay more in a given state ω the tighter the period 0 budget constraint as captured by ν and the higher the productivity shock A_1^ω.)

Define the amount of funds raised in period 0 for a given $\nu \in (0, \nu^{\max}]$ as

$$R_0(\nu) = E\left[m_1(b_1(\nu; A_1^\omega)) \cdot b_1(\nu; A_1^\omega)\right]$$

The function $R_0 : (0, \nu^{\max}] \to (-e, R_0^{\max}]$ is continuous and, by assumption A.3, strictly increasing. For $\nu = \nu^{\max}$ as defined in equation (17) it reaches its maximum, which we denote as

$$R_0^{\max} = R_0(\nu^{\max}) > 0 \tag{18}$$

By assumption A.1 the initial investment requirement of bankers is low enough to fall within the defined range of the function $\alpha t_1 \in (0, R_0^{\max}]$. The period 0 budget constraint of bankers can be written as

$$R_0(\nu) = \alpha t_1 \tag{19}$$

The strict monotonicity of $R_0(\cdot)$ implies that there is a unique solution $\nu^* \in (0, \nu^{\max}]$ that satisfies this equation. Given the equilibrium ν^*, the optimal borrowing choices of bankers are $b_1(\nu^*; A_1^\omega)$. All other variables follow.

Proposition 1 (Decentralized Equilibrium Under Loose Constraints) *If $\alpha t_1 \in [0, \hat{R}_0]$, bankers contract a constant repayment across all states of nature $b_1^\omega = b_1^{unc}(\nu^*)$ $\forall \omega$, where the threshold $\hat{R}_0$ is defined as*

$$\hat{R}_0 = m_1(A^{\min}t_1)A^{\min}t_1 = \frac{u'\left(e + A^{\min}t_1\right)}{u'\left(e - \alpha t_1\right)} \cdot A^{\min}t_1 \tag{20}$$

If bankers are unconstrained across all states of nature, they carry all risk and provide households with a fixed repayment, corresponding to a risk-free bond. This case corresponds to the lower line $b_1^{unc}(\nu_{II}^*)$ in figure 3.

The ceiling $\hat{R}_0$ in the proposition captures the amount of period 0 finance that is raised by the largest possible fixed payment of bankers $A^{\min}t_1$ that avoids binding constraints and fire sales in the lowest state of nature $A^{\min}$. Note that $\hat{R}_0$ is higher the greater the

[6] As we discuss in appendix A.1, it would be straightforward to generalize our focus somewhat, but our main results would be unaffected.

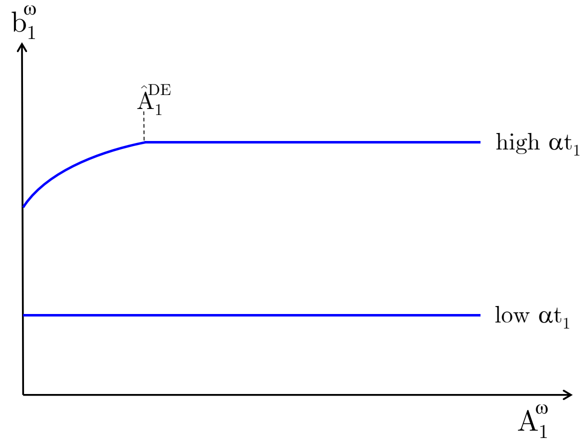

Figure 3: Contingent repayment b_1^ω for high/low initial investment requirement

minimum period 1 return $A_1^{\min}$ and the higher the elasticity of substitution of first generation households, since a higher elasticity of substitution implies that households require less compensation to accept an unsmooth consumption profile.

Proposition 2 (Decentralized Equilibrium Under Partially Binding Constraints) *Otherwise, if $\alpha t_1 \in (\hat{R}_0, R_0^{\max}]$, then the equilibrium is characterized by a threshold $\hat{A}_1 > A^{\min}$ such that:*

- *For $A_1^\omega \geq \hat{A}_1$, bankers contract a fixed repayment $b_1^\omega = b_1^{unc}(\nu^*) = \hat{A}_1 t_1 \; \forall \omega$. Their consumption is $c_{1,b}^\omega = (A_1^\omega - \hat{A}_1)t_1$, i.e. they absorb all output risk beyond the threshold $\hat{A}_1$. They engage in no fire sales $f_1^\omega = 0$.*
- *For $A_1^\omega < \hat{A}_1$, bankers reduce their period 1 repayment compared to unconstrained states $b_1^\omega = b_1^{\omega,con}(\nu^*) < \hat{A}_1 t_1$ and engage in positive fire sales $f_1^\omega > 0$. Their period 1 consumption is zero $c_{1,h} = 0$.*

The resulting payment profile is similar to a defaultable bond – bankers pay a fixed repayment in high states of nature – if their output is sufficient to cover the fixed repayment – and the entire output plus receipts from fire sales in low states of nature when they are in financial distress. This case is illustrated by the upper line $b_1^\omega(\nu_I^*)$ in figure 3.

Ex ante, reducing the repayment b_1^ω or engaging in fire sales f_1^ω in low states of nature are two alternative costly ways of obtaining liquidity: when bankers engage in fire-sales, asset prices decline so that their proceeds are less than the marginal product that they could have earned on the assets. Similarly, the cheapest way for bankers to raise a given amount of finance from first-generation households is to promise a constant payment across all states of nature since households are risk-averse. If bankers reduce their promised repayments b_1^ω

in low states so as to insure themselves and increase b_1^ω in high states of nature to make up for it, the total interest bill rises. Bankers pick their portfolios such that the relative costs of the two forms of raising liquidity are equal from their private perspective.

4 Welfare Analysis

In this section, we analyze welfare in the decentralized equilibrium and characterize Pareto-improving allocations that could be chosen by a planner in the described economy.

4.1 Effects of Marginal Reduction in Fire Sales

Let us first analyze the scope for Pareto improvements in the economy by considering the welfare effects of a marginal reallocation of security issuance that aims at reducing fire sales. Suppose the economy is in a decentralized equilibrium with partially binding constraints (as described in proposition 2). Assume there are two states of nature $\omega, \psi \in \Omega$ of equal probability density where the period 1 equilibrium in state ω exhibits binding constraints and in state ψ loose constraints. Consider a planner in period 0 who reduces security issuance of bankers in the constrained state ω by an infinitesimal amount db_1^ω in period 0 while holding the prices of Arrow securities constant. In order to satisfy the period 0 budget constraint of bankers, the planner increases security issuance conditional on the unconstrained state ψ by $db_1^\psi = -db_1^\omega \frac{m_1^\omega}{m_1^\psi} > 0$. By the envelope theorem, the change in utility of first-generation households is second-order because they were previously at their optimum.

Remark: If we allowed period 0 prices m_1^ω and m_1^ψ of Arrow securities to adjust, then there would in general be a redistribution of welfare from bankers to households. The planner could undo this by providing a lump-sum transfer from households to bankers.

In period 1, bankers have db_1^ω more liquid resources in state ω. An atomistic agent who takes asset prices as given would anticipate that this allows him to reduce fire sales by

$$\frac{\partial f_1}{\partial b_1^\omega} = \frac{1}{q_1}$$

However, in general equilibrium, the reduction in fire sales pushes up asset prices and leads to an amplified decline in fire sales by

$$\frac{df_1}{db_1^\omega} = \frac{1}{q_1} \cdot \frac{1}{1 - \eta_{qf}}$$

as we had captured in equation (11). Employing these assets in production allows bankers to consume $\frac{\bar{A}_2}{q_1^\omega} \cdot \frac{db_1^\omega}{1-\eta_{qf}} = \frac{\mu^\omega db_1^\omega}{1-\eta_{qf}}$ more in period 2 of state ω.

Similarly, in state ψ the increase in the promised repayment requires bankers to reduce period 1 consumption by $db_1^\psi = -\frac{m_1^\omega}{m_1^\psi} db_1^\omega = -\mu^\omega db_1^\omega$, since bankers in the decentralized equilibrium choose repayments such that $\frac{m_1^\omega}{m_1^\psi} = \frac{\mu^\omega}{\mu^\psi}$ and since $\mu^\psi = 1$ in unconstrained states. The net change in the banker's state utility from the planner's reallocation over states ω and ψ is

$$\frac{d\left(V^\omega + V^\psi\right)}{db_1^\omega} = \frac{\mu^\omega}{1-\eta_{qf}} - \mu^\omega = \frac{\eta_{qf}}{1-\eta_{qf}} \cdot \mu^\omega$$

Second generation households are unaffected in state ψ, but pay $dq_1^\omega = \frac{\partial q}{\partial f} df_1^\omega = \frac{1}{f_1^\omega} \cdot \frac{\eta_{qf}}{1-\eta_{qf}} db_1^\omega$ more per unit of asset purchased, implying a change in their utility in state ω of

$$\frac{dW^\omega}{db_1^\omega} = -f_1^\omega \cdot \frac{dq_1^\omega}{db_1^\omega} = -\frac{\eta_{qf}}{1-\eta_{qf}}$$

(Since second generation households purchase assets up to the point where $F'(f_1^\omega) = q_1^\omega$, the welfare effects of the reduction in the quantity of assets used in production are second order.)

Since $\mu^\omega > 1$, the planner could transfer $\frac{\eta_{qf}}{1-\eta_{qf}} db_1^\omega$ from bankers to second generation households in the unconstrained state ψ to compensate them for the reallocation in state ω. This leaves households indifferent and achieves a first order welfare gain for bankers. The described reallocation therefore constitutes a Pareto improvement.

4.2 Planning Problem

Before analyzing the problem more fully, let us note that a planner who has the ability to arbitrarily redistribute funds between agents in the economy could implement the first-best solution to our probem, in which bankers are always allocated all productive assets in the economy since they have a superior production technology. In such a setup financial constraints would be irrelevant and the solution is trivial.

In the remainder of this section, we therefore assume that the planner cannot make transfers to bankers in states of nature when they experience binding constraints. However, we assume that the planner instead has a regulatory tool to determine the financing and risk-taking decisions b_1^ω of bankers in period 0. In addition, she has the means to engage in lump-sum transfers to ensure that efficiency gains are spread around the economy such that all agents in the economy are (weakly) better off.

We formulate the planner's problem as

$$\max_{\{T_0, b_1^\omega, c_{1,b}^\omega, f_1^\omega, T_1^\omega\}} E\left[c_{1,b}^\omega + \bar{A}_2(t_1 - f_1^\omega)\right] \quad \text{s.t.} \quad \alpha t_1 = E[m_1^\omega b_1^\omega] - T_0 \tag{21}$$

$$c_{1,b}^\omega = A_1^\omega t_1^\omega - b_1^\omega + q_1^\omega f_1^\omega - T_1^\omega \geq 0$$

$$T_1^\omega \geq 0,\ m_1^\omega = \frac{u'(e+b_1^\omega)}{u'(e-\alpha t_1)},\ q_1^\omega = F'(f_1^\omega)\ \forall\ \omega$$

$$U \geq U^{DE}, W \geq W^{DE}$$

where T_0 and T_1^ω represent compensatory transfers from bankers to first-generation and second generation households in periods 0 and 1 respectively. Note that we impose a constraint $T_1^\omega \geq 0$, which ensures that the planner uses transfers in period 1 solely for compensatory reasons – to transfer resources away from bankers to second generation households – and not to relax the financial constraints of bankers by transferring resource to them. U^{DE} and W^{DE} are the utility levels of first and second generation households in the decentralized equilibrium of the economy. (In this section, we denote variables that refer to the decentralized equilibrium with superscript 'DE' and – in case of ambiguity – variables that refer to the social planner's allocation by 'SP'.)

Backward Induction: Optimal Period 1 Solution

We first focus on a planner who takes as given the net liquid assets a of bankers in a given state of nature of period 1, after first-generation households have been repaid. The planner maximizes total surplus of bankers and second-generation households over periods 1 and 2 while ensuring that the utility of the latter satisfies $W \geq W^{DE}$. Since both bankers and second generation households have linear utility, the planner can transfer resources between the two at a rate of one for one whenever financial constraints are loose. We simplify the problem in the following way:

Lemma 1 *A planner in period 1 can focus on maximizing total surplus $S(a)$ while compensating second generation households by providing lump-sum transfers T_1^ω in unconstrained states that satisfy*

$$E[T_1^\omega] = E\left[F(f_1^{\omega,DE}) - q_1^{\omega,DE} f_1^{\omega,DE} - F(f_1^{\omega,SP}) + q_1^{\omega,SP} f_1^{\omega,SP}\right] \quad \textit{where} \quad T_1^\omega \geq \max\{0, a^\omega\} \tag{22}$$

This is feasible as long as the required transfer is less than the total liquid assets of bankers across all unconstrained states $E[\max\{0, a^\omega\}]$.

Equation (22) captures that the transfer needs to make up for the loss in utility that second generation households suffer because of reductions in their profits $F(f_1) - q_1 f_1$ from processing fire-sales. Given the linear utility functions of both agents, the precise allocation of transfers across states of nature is indeterminate – the planner can engage in compensatory transfers in any unconstrained states as long as the magnitude of the required transfer does not exceed the expected positive liquid asset holdings of bankers $E[\max\{0, a^\omega\}]$. In the following, we will assume that this condition is met.[7]

Remark: An alternative specification would be to require that the planner's transfer is uncontingent $T_1^\omega = \bar{T}_1$. This would still allow the planner to implement Pareto improvements, but it would imply that bankers are required to make some transfers in constrained states of nature even if they have resources available in unconstrained states, which is inefficient.

Using the lemma, we express the planner's welfare in a given state of period 1 as the sum of the utility of bankers and second generation households. The solution to this problem closely reflects that of the decentralized period 1 problem of bankers (10) – the planner finds it optimal to engage in the minimum amount of fire-sales possible. For $a \geq 0$ there are no fire sales ($f_1 = 0$) and total social surplus amounts to $a + \bar{A}_2 t_1 + 2e$, where the last two terms are constants. For $a < 0$, a quantity $f_1 = f(-a)$ of assets are fire-sold at price $q(-a)$ and produce output of $F(f_1)$ instead of $\bar{A}_2 f_1$. We summarize the planner's social surplus as a function of the liquid assets a held by bankers as

$$S(a) = a + F(f(-a)) - \bar{A}_2 f(-a) + \text{const} \tag{23}$$

The marginal social valuation of liquidity as perceived by the planner is

$$\mu^{SP} = S'(a) = 1 + \left[\bar{A}_2 - F'(f_1)\right] \cdot f'(-a) = \frac{\bar{A}_2/q_1 - \eta_{qf}}{1 - \eta_{qf}} \tag{24}$$

where we substituted $f' = \frac{1}{q_1} \cdot \frac{1}{1-\eta_{qf}}$ from (11). In comparing the marginal valuation of liquidity of decentralized bankers and of the planner, we find

Proposition 3 (Valuation of Liquidity) *For $a \geq 0$, financing constraints are loose and the valuation of liquidity of decentralized bankers and the planner coincide $\mu^{SP} = \mu^{DE} = 1$.*

For $a < 0$, financing constraints on bankers are binding, and the planner values liquidity more highly than decentralized agents $\mu^{SP} > \mu^{DE} > 1$. The private undervaluation of liquidity is more severe the closer νqf to 1.

[7] If the expected value of the required transfer is greater than $E[\max\{0, a^\omega\}]$, then the planner would instruct bankers to raise a minimum of $\bar{s}$ through fire sales in each state of nature and transfer their liquidity to second generation households, i.e. $T_1^\omega = \max\{0, a^\omega + \bar{s}\}$, where $\bar{s}$ is chosen such that resulting revenue raised satisfies equation (22).

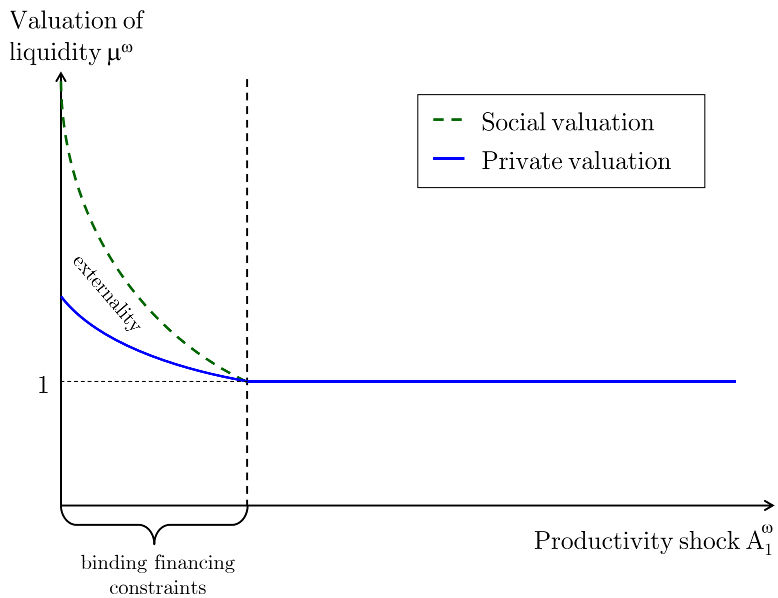

Figure 4: Private and Social Valuation of Liquidity

Proof. The inequality follows from $\mu^{DE} = \frac{\bar{A}_2}{q_1} \geq 1$ in the numerator of (24) and from $\eta_{qf} < 1$, which holds by assumption A.3. ■

Remark: A planner internalizes that a decline in asset prices leads to financial amplification since it reduces the amount of liquidity that bankers can raise from their sales of each unit of the assets. This pecuniary externality reduces the efficiency of the distribution of capital. By contrast, decentralized bankers take asset prices as given since they realize that the behavior of an atomistic agent has only an infinitesimal effect on asset prices. This is the basis of the inefficiency result in our paper.

Figure 4 schematically depicts the valuation of liquidity of decentralized agents and the planner across different states of nature assuming a fixed debt level $\bar{b}_1$. In normal times when constraints are loose, the two coincide and equal 1. When financing constraints are binding, $\mu^{\omega,SP} > \mu^{\omega,DE}$ since the planner internalizes that higher liquidity would mitigate the downward spiral in asset prices and production.

Optimal Period 0 Financing Decisions

Let us now turn to the implications of this difference in the valuation of liquidity between planner and decentralized agents for the period 0 allocation of Arrow securities. Having characterized the function $S(a)$, the planner's problem (21) can be reformulated as

$$\max_{\{T_0,b_1^\omega\}} E\left\{S(A_1^\omega t_1 - b_1^\omega)\right\} \quad \text{s.t.} \quad U \geq U^{DE} \tag{25}$$

$$E[m_1^\omega b_1^\omega] = \alpha t_1 + T_0$$

$$m_1^\omega = \frac{u'(e+b_1^\omega)}{u'(e-\alpha t_1)}$$

The planner can split the problem into two steps: first she optimizes

$$\max_{\{b_1^\omega\}} E\left\{S(A_1^\omega t_1 - b_1^\omega)\right\} \qquad \text{s.t.} \qquad U = u(e-\alpha t_1) + E\left[u(e+b_1^\omega)\right] \geq U^{DE} \tag{26}$$

Taking the resulting allocation of Arrow securities $\{b_1^\omega\}$, she sets

$$T_0 = E[m_1^\omega b_1^\omega] - \alpha t_1 = E\left[\frac{u'(e+b_1^\omega)}{u'(e-\alpha t_1)} \cdot b_1^\omega\right] - \alpha t_1 \tag{27}$$

so as to satisfy the remaining two constraints in problem (25).

Normalizing the constraint $U \geq U^{DE}$ by dividing through the constant $u'(e-\alpha t_1)$ and assigning the multiplier ν^{SP}, the first order condition of (26) is

$$S'(a^\omega) = \nu^{SP} m_1^\omega$$

Substituting for $S'(a^\omega) = \mu^{\omega,SP}$ and m_1^ω we obtain

$$\left(\frac{\bar{A}_2}{q(b_1 - A_1 t_1)} - \eta_{qf}\right) \Big/ (1-\eta_{qf}) = \nu \cdot \frac{u'(e+b_1)}{u'(e-\alpha t_1)} \tag{28}$$

This optimality condition has the same format as the decentralized optimality condition (14) and can be solved in a similar manner. In particular, the equilibrium condition (28) defines a function $b_1^{SP} : (0, \nu^{\max,SP}] \times [A^{\min}, A^{\max}] \to (-e, A^{\min} t_1 + s^{max}]$ that satisfies $\partial b_1^{SP}/\partial \nu^{SP} > 0$ and $\partial b_1^{SP}/\partial A_1 \geq 0$. The maximum value of ν^{SP} for which the function is defined is $\nu^{SP,\max} = [\bar{A}_2/q^{\min} - \eta_{qf}]/(1-\eta_{qf}) \cdot \frac{u'(e-\alpha t_1)}{u'(e+A_1^{\min} t_1 + s^{\max})} > \nu^{DE,\max}$, since the planner perceives a higher shadow cost of fire-selling all of the banker's assets than decentralized bankers.

Observe that expression (28) reduces to the optimality condition of decentralized bankers (14) when there are no fire sales and $\frac{\bar{A}_2}{q(b_1 - A_1 t_1)} = 1$. In comparing the planner's repayment function $b_1^{SP}(\cdot)$ and the decentralized repayment function $b_1^{DE}(\cdot)$ we find

Lemma 2 *For a given pair $(\nu, A_1) \in (0, \nu^{\max,DE}] \times [A^{\min}, A^{\max}]$,*

$$\left.b_1^{SP}(\nu; A_1) = b_1^{DE}(\nu; A_1) = b_1^{unc}(\nu)\ \right\} \text{ for } \nu \leq \hat{\nu}(A_1)$$

$$\left.\begin{array}{r} b_1^{SP}(\nu; A_1) < b_1^{DE}(\nu; A_1) \\ \text{and} \quad \frac{db_1^{SP}(A_1,\nu)}{dA_1} > \frac{db_1^{DE}(A_1,\nu)}{dA_1} > 0 \end{array}\right\} \text{ for } \nu > \hat{\nu}(A_1)$$

A detailed proof is given in appendix A.4. Intuitively, if ν is low enough that the corresponding repayment can be made without incurring binding constraints, the planner and decentralized agents value liquidity equally and repay identical amounts. If the repayment requires fire-sales, the planner values liquidity more highly and repays less for a given shadow cost of raising period 0 funds. Furthermore, as A_1^ω decreases, the planner decreases repayments more rapidly than decentralized agents.

Proposition 4 (Planner's Solution Under Loose Constraints) *If $\alpha t_1 \in [0, \hat{R}_0]$, i.e. if the decentralized equilibrium exhibits loose constraints in all states of nature, then the decentralized equilibrium coincides with the planner's optimal allocation.*

Proof. Consider an economy that satisfies condition (20) so that the decentralized equilibrium exhibits loose constraints across all states of nature. In period 1, $a^{\omega,DE} \geq 0 \ \forall\omega$, therefore both decentralized agents and the banker find it optimal to not engage in fire sales, but to allocate the banker's net liquid assets to consumption $c_{1,b}^\omega = a^\omega$. Furthermore, the utility of second generation households is at the level that they receive in the decentralized equilibrium, so the planner does not need to compensate them and $T_1^\omega = 0$.

Note that the marginal valuations of period 1 liquidity of decentralized bankers and of the planner coincide for this allocation, i.e. $\mu^{\omega,DE} = \mu^{\omega,SP} = 1$ for net liquid assets $a^{\omega,DE}$. Therefore the optimality conditions on b_1^ω in the decentralized equilibrium (14) and in the planner's optimum (28) coincide and the decentralized allocation satisfies the planner's optimality conditions for $\nu^{*SP} = \nu^{*DE}$. Note that in the given allocation, household utility is at the level of the decentralized equilibrium and the planner finds it optimal to set the period 0 transfer to $T_0 = 0$. ∎

Proposition 5 (Planner's Solution Under Partially Binding Constraints) *If $\alpha t_1 \in (\hat{R}_0, R_0^{\max}]$, i.e. if the decentralized equilibrium exhibits binding constraints in some states of nature, then the planner achieves a Pareto improvement over the decentralized equilibrium. The planner's allocation is described by a shadow price $\nu^{*SP} > \nu^{*DE}$ and a productivity threshold for binding constraints $\hat{A}_1^{SP} > \hat{A}_1^{DE}$ such that:*

- *For $A_1^\omega \geq \hat{A}_1^{SP}$, the planner contracts a higher fixed repayment $b_1^\omega = b_1^{unc}(\nu^{*SP}) = \hat{A}_1^{SP} t_1$ than decentralized bankers and there are no fire sales.*

- *For $A_1^\omega < \hat{A}_1^{SP}$, the planner chooses repayments below $\hat{A}_1^{SP} t_1$ and makes smaller payments than decentralized bankers $b_1^{\omega,SP} < b_1^{\omega,DE}$ in the lowest states of nature.*

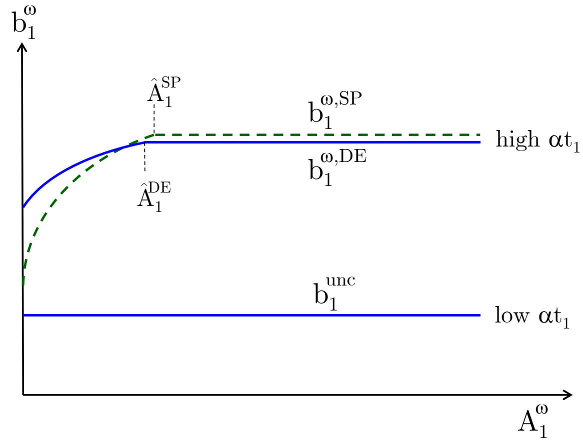

Figure 5: Comparison of repayment b_1^ω for decentralized bankers and planner

Proof. Let us rewrite the constraint $U^{SP} \geq U^{DE}$ guaranteeing that first generation households are not worse off in the planner's allocation:

$$E[u(e + b_1^{SP}(\nu; A_1))] \geq E[u(e + b_1^{DE}(\nu^{*DE}; A_1))] \tag{29}$$

By lemma 2 the inequality would be violated for $\nu = \nu^{*DE}$. Since the left-hand side of the inequality is strictly increasing in ν, the constraint is satisfied with equality for the optimal $\nu^{*SP} > \nu^{*DE}$. It follows immediately that in unconstrained states, $b_1^{\omega,SP} = b_1^{unc}(\nu^{*SP}; A_1^\omega) > b_1^{unc}(\nu^{*DE}; A_1^\omega)$ and that the threshold for binding constraints $\hat{A}_1^{SP} = b_1^{unc}(\nu^{*SP})/t_1$ rises.

Since equation (29) must hold with equality, higher repayments in all unconstrained states imply that there must be lower repayments in some constrained states $b_1^{\omega,SP} < b_1^{\omega,DE}$. The third equation of lemma 2 reveals that $b_1^{SP}(\cdot)$ rises faster in A_1 than $b_1^{DE}(\cdot)$ for a given ν. It follows that the planner repays less than decentralized agents in those states of nature with the lowest realizations of productivity A_1^ω, i.e. in which there are the greatest fire sales. The payments $\{b_1^{\omega,SP}\}$ chosen by the planner are more expensive in period 0 as captured by $E[m_1(b_1^{\omega,SP})b_1^{\omega,SP}]$ than those chosen in the decentralized equilibrium – in fact this price signal is the reason why decentralized agents purchase less insurance against constrained states. The planner's allocation satisfies the optimization problem (25) if she chooses a positive transfer $T_0 = E[m_1(b_1^{\omega,SP})b_1^{\omega,SP}] - \alpha t_1$ as determined by equation (27). ■

Figure 5 illustrates the differences between the repayments contracted by decentralized bankers and by the planner graphically. For a low initial investment requirement bankers make a fixed repayment b_1^{unc} and do not experience binding constraints – the two equilibria coincide. For a high initial investment requirement the planner repays more in unconstrained states and less in most constrained states of nature compared to bankers.

Remark 1: The planner reallocates period 1 repayments b_1^ω from strongly constrained states to unconstrained states of nature in order to reduce socially inefficient fire sales and output declines. The planner therefore purchases more 'insurance' against low output states that exhibit binding constraints. In practice, this could be interpreted as substituting debt finance by other, more contingent forms of finance.

Remark 2: In the described economy, the aggregate productivity shock A_1^ω constitutes systematic risk. Whenever financial constraints are binding, amplification effects are triggered and the shock triggers systemic risk, which bankers would like to insure against. First generation households are risk-averse and require compensation for taking on this risk, and the decentralized equilibrium is therefore characterized by the privately optimal trade-off between the cost of consumption volatility for households and the efficiency cost of fire-sales for bankers. However, since decentralized bankers internalize only part of the social benefit of insuring against fire-sales, they leave themselves exposed to too much systemic risk. As a result, the economy is characterized by an excessive extent of financial amplification and excessive declines in asset prices and output in low states of nature.

We emphasized in propositions 2 and 5 that systemic risk and socially excessive fire sales arise whenever the initial investment requirement of bankers is so high that it is optimal to incur some fire sales in low states of nature ($\alpha t_1 > \hat{R}_0$), as captured by condition (20). Let us discuss the circumstances that determine whether this condition is likely to be satisfied. Assume for simplicity that the utility function of first-generation households exhibits constant relative risk aversion θ.

Corollary 1 (Incidence of Systemic Risk) *The economy is more prone to systemic risk, i.e. condition (20) is more likely to be violated,*

- *the higher the initial financing requirement αt_1 of bankers*
- *the lower the endowment of first-generation households*
- *the lower the minimum period 1 return of bankers $A^{\min} t_1$*
- *the greater the relative risk aversion of households θ.*

5 Applications

Having analytically characterized the externality that is the subject of this paper, we turn our attention to a number of applications, including the effects of anticipated government bailouts, the implementation of the planner's solution via macroprudential regulation, and the suboptimal incentives for bankers to raise new capital in states of systemic amplification.

5.1 Bailout Neutrality

When binding constraints and financial amplification in an economy are triggered, government authorities find it ex post optimal to intervene by providing lump-sum transfers ('bailouts') to constrained bankers. This allows them to mitigate the amplification effects and the associated decline in asset prices and output. This section shows that if such bailout transfers are anticipated, decentralized bankers will find it optimal to fully undo them.

Assume that a government commits to a state-contingent period 1 lump-sum transfer Z^{ω} that provides a bailout $Z^{\omega} > 0$ to bankers in states of nature in which they experience binding constraints. If bankers are unconstrained, government levys a fee $Z^{\omega} < 0$ on them so as to make the policy revenue-neutral in expectation. Assume that the government buys the respective state contingent securities from first generation households at time 0 and distributes the transfers to bankers in period 1 after the productivity shock is realized. The assumption of revenue neutrality implies that the total expenditure on such securities in period 0 is

$$E\left[m_1^{\omega} Z^{\omega}\right] = 0$$

If we add these transfers to the optimization problems of bankers and first-generation households, their first order conditions are unaffected: decentralized bankers chose their equilibrium allocations on the basis of an optimal tradeoff of risk versus return. If they receive one more dollar in period 1 of a given state ω, they will sell one more bond contingent on that state so as to restore their privately optimal equilibrium.

Proposition 6 (Bailout Neutrality) *An anticipated state-contingent lump sum transfer Z^{ω} to bankers that satisfies $E[m_1^{\omega} Z^{\omega}]$ will be fully undone by optimizing bankers.*

Specifically, the private sales of state-contingent securities of bankers under such a transfer will satisfy

$$b_1^{\omega,Z} = b_1^{\omega,DE} + Z^{\omega}$$

This implies that – after the transfer has occured – all other allocations and prices in the economy are identical to those of the decentralized equilibrium. Our finding represents a state-contingent form of Ricardian equivalence (Barro, 1974). Bankers see through the fiscal veil and add up their private budget constraint and the government's transfers Z^{ω} when determining their optimal decisions.

Remark 1: The proposition also suggests circumstances under which bailouts may be effective. This may be the case if (a) they are unanticipated or (b) if bankers are prevented from undoing the transfers, either because of regulatory constraints or because the state-contingent markets required for this do not exist.

Remark 2: Transfers in constrained states that were anticipated but that end up not taking place can have strongly negative effects, since any exogenous change Δa to the liquidity position of bankers under binding constraints is amplified. The expectation of a bailout leads bankers to take on larger risks than what is privately optimal in the absence of government intervention; their liquidity position after the shock is therefore below what is privately optimal, and by implication even further below what is socially optimal.

Our bailout neutrality result captures a stark version of what is sometimes described as the 'moral hazard' introduced by the anticipation of government bailouts. In the described setting, private bankers find it optimal to engage in socially excessive risk-taking if there are some states of nature in which financial constraints are binding. Even if bailouts are lump-sum and do not distort the marginal incentives of bankers as captured by their optimality conditions, they find it optimal to undo them in order to return to their privately optimal allocations. This suggests that in the given setting, lump sum transfers cannot improve upon the allocations of the decentralized equilibrium in an ex-ante sense.

5.2 Macroprudential Regulation

In this section, we examine how a planner can implement Pareto superior allocations by resorting to a tax on risk-taking that brings the private costs of risk-taking in line with the social cost. We call this measure "macroprudential regulation" because it closely captures what the Bank for International Settlements defines as the macroprudential approach to regulation (see e.g. Borio, 2003): it is designed to limit system-wide financial distress that stems from the correlated exposure of financial institutions and to avoid the resulting real output losses in the economy.

Definition 1 (Externality Kernel) *We define the externality kernel τ^ω of bankers as the difference between the private valuation and the planner's social valuation of period 1 liquidity*

$$\tau^\omega = \mu^{\omega,SP} - \mu^{\omega,DE} \tag{30}$$

This captures the un-internalized social cost of financially constrained bankers making a payment of one dollar in state ω. Following proposition 3, the externality kernel is zero in unconstrained states and positive in constrained states. Since the productivity shock A_1^ω is the only source of uncertainty in the model and since lower realizations of productivity are associated with tighter constraints, we find $\mathrm{Cov}(\tau^\omega, A_1^\omega) < 0$ whenever there are states with binding constraints.

We observed from equation (28) that the decentralized solution and the planner's allocation in period 0 differ only to the extent that their respective valuations of liquidity differ in constrained states of nature, with the difference captured by the externality kernel. By raising the private valuation to the social valuation of liqudity in all states of nature, a state-contingent tax on the issuance of Arrow securities allows the planner to replicate this allocation in a market setting.

Proposition 7 (Macroprudential Taxation) *A planner who imposes a state-contingent proportional tax τ^ω on the issuance of Arrow securities b_1^ω in period 0 restores constrained social efficiency. Compensatory transfers T_0 and T_1^ω as defined in (22) and (27) ensure that the resulting allocation constitutes a Pareto improvement.*

To move our discussion from Arrow securities to more complex financial securities, assume that the economy is in the equilibrium described in the previous proposition and consider an atomistic banker who sells a unit of a financial claim in period 0 that has a state-contingent net payoff profile X^ω in period 1. Such a claim can be viewed as a collection of Arrow securities with weights X^ω. It can be interpreted alternatively as an investment in risky assets that is financed by uncontingent debt and yields a net payoff of X^ω.

Corollary 2 (Taxation of Complex Securities) *The externalities imposed by a financial payoff X^ω are $E[\tau^\omega X^\omega]$. The optimal period 0 tax that induces a banker to internalize the full social cost of holding security X^ω is*

$$\tau_X^* = E[\tau^\omega X^\omega] = E[\tau^\omega]E[X^\omega] + Cov\left(\tau^\omega, X^\omega\right) \tag{31}$$

To gain some intuition, let us compare the magnitude of the externalities imposed by a number of securities with different payoff profiles. Figure 6 schematically depicts a few examples. First, for an uncontingent bond with a face value of one dollar, the payoffs are $X^\omega = 1$ in all constrained and unconstrained states of nature. The optimal tax on such a bond is $E[\tau^\omega]$.

Next consider a risky security with an expected payoff $E[X^\omega]$ of one dollar. The externality imposed by such a security is $E[\tau^\omega] + Cov\left(\tau^\omega, X^\omega\right)$. If the payoff X^ω of a security and the externality kernel have positive covariance, then the security imposes larger externalities and embodies more systemic risk than an uncontingent bond, and therefore calls for greater macroprudential taxation. A stark example would be a credit default swap, which is likely to require large payouts precisely in times of financial turmoil, i.e. when economy-wide financial constraints bind and when the externality kernel τ^ω is high.[8]

[8]The payoff profile drawn in figure 6 is not based on a specific analytical example but illustrates the assumption that defaults in the economy occur when the banking sector as a whole experience binding constraints.

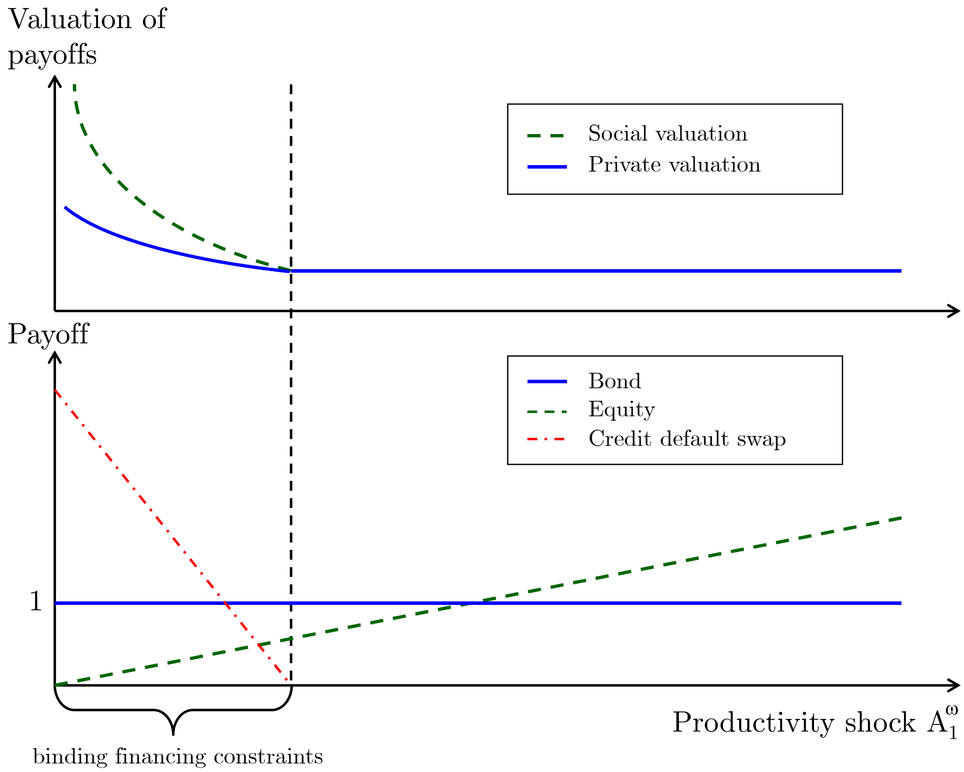

Figure 6: Schematic payoff profile of uncontingent bond, equity and credit default swap

On the other hand, the more negatively the payoffs X^{ω} of a security covary with the externality kernel, the more insurance the state-contingent payoff provides, the smaller the externality and the lower the optimal tax. An example would be if bankers sell equity paying dividends that are linear in the state of productivity A_1^{ω}, which is by construction negatively correlated with τ^{ω}. Note that the optimal tax τ_X^* is not bounded at zero. If a security offers sufficient systemic insurance benefits, i.e. it provides positive payoffs to bankers precisely when they are constrained and subject to financial amplification effects, then it imposes a positive systemic externality and should be subject to a subsidy, or a reduction in the capital that banks are required to hold. An example would be a credit default swap that shifts systemic risk to agents outside of the financial system who are not subject to financial constraints.

Equivalent Capital Adequacy Requirements While we have formulated our policy measures in terms of taxes, banking regulations typically take the form of capital adequacy requirements, which have tax-like effects since bank capital is costly. If the opportunity cost of holding one dollar of capital is δ for a bank, then a tax τ_X^* is equivalent to a capital requirement of τ_X^*/δ.

Macro- vs. Micro-Prudential Regulation Equation (31) captures that what matters for macroprudential regulation is not the general riskiness inherent in a security, as described e.g. by the variance of its payoff, but rather the correlation with systemic risk, as described by its covariance with the externality kernel τ^ω. This is commonly viewed to be an important feature of macroprudential regulation (Borio, 2003).

Leverage Leverage multiplies gains or losses by using uncontingent debt to increase the amount invested in a risky security. For example, if a risky investment with payoff X^ω in period 1 is leveraged by a factor $\alpha > 1$, then $(\alpha - 1)$ units are financed by debt and the total payoff is

$$\alpha X^\omega - (\alpha - 1)\frac{E[m_1^\omega X^\omega]}{E[m_1^\omega]}$$

where $E[m_1^\omega X^\omega]$ is the period 0 price of the investment and $\frac{1}{E[m_1^\omega]}$ is the risk-free interest rate. This amounts to an increase in the dispersion of the total payoff by a factor α, which raises its covariance with the externality kernel in equation (31) equiproportionally and increases the externalities of the investment accordingly.

Reach of Regulation Our theory also offers insights into the question about the reach of regulation: macroprudential regulation should apply to any financial market participant who might potentially be forced to engage in fire-sales during periods of system-wide amplification effects, since a rational private actor would not internalize the price effects of such sales and the externalities on the financing constraints of other market participants. This includes hedge funds and other actors in the so-called "shadow financial system."

Socially Risk-Neutral Probabilities Pricing kernels can alternatively be represented as a risk-neutral probability measure that weighs states against which agents are risk-averse more highly. We can apply a similar transformation to the social planner's pricing kernel. If regulators instruct banks to employ the regulator's risk-neutral probabilities in their risk management systems, the externality that is the topic of this paper would be alleviated.

Analytically the socially risk-neutral probabilities can be obtained from the standard formula

$$g_{rn}(\omega) = \frac{g(\omega)\mu^{SP,\omega}}{E[\mu^{SP,\omega}]}$$

and the social value of a payoff X^ω can be expressed as $E_{rn}[X^\omega]$, where $E_{rn}[\cdot]$ represents the expectations operator under the socially risk-neutral probability measure defined by f_{rn}. This measure weighs states of the world in which amplification effects arise more highly than what would be indicated by a traditional 'privately' risk-neutral probability measure, which in turn assigns more weight to such states than the objective probability of that state.

Market Discipline It has been argued that transparency requirements in conjunction with the market discipline embodied by pillar 3 of the Basel accord would induce banks to optimally smooth their capital position throughout the business cycle (see e.g. Gordy and Howells, 2006, for a discussion of this argument). In the absence of regulations of systemic externalities, our analysis suggests that markets would actually punish prudent banks that behave socially responsibly and would reward banks that take on socially excessive risks, since maximizing shareholder value involves excessive risk-taking.

5.3 Raising New Capital

Next we extend our model of the previous sections to study the incentives for bankers to raise capital. Suppose we introduce an audit technology that gives bankers a way around the pledgeability problem for period 2 payoffs. Specifically, assume that second-generation households can take ownership of a fraction γ of bankers' period 2 returns as long as they pay a convex auditing cost $c(\gamma)$ in period 1, where $c(0) = c'(0) = 0$ and $c'(\gamma), c''(\gamma) > 0$ for $\gamma > 0$. Since they are risk-neutral, they are willing to provide $\gamma\bar{A}_2(t_1 - f_1) - c(\gamma)$ in return for their ownership share.

The resulting version of the period 1 problem (10) of a decentralized banker is

$$V(a) = \max_{\{c_{1,b}, f_1, \gamma\}} c_{1,b} + (1-\gamma)\bar{A}_2(t_1 - f_1) - \mu\left[c_{1,b} - a - q_1 f_1 - \gamma\bar{A}_2(t_1 - f_1) + c(\gamma)\right] + \lambda c_{1,b}$$

Similarly, total period 1 surplus $S(a)$ can be formulated by modifying the planner's problem (23) analogously. For both private bankers and the planner, the optimality condition with respect to γ is

$$1 = \mu\left[1 - \frac{c'(\gamma)}{\bar{A}_2(t_1 - f_1)}\right]$$

As we observed in proposition 3, $\mu^{DE} = \mu^{SP} = 1$ if the economy is unconstrained, implying that bankers and the planner will not raise new capital in period 1 and $\gamma = 0$. This is because raising new equity is not useful to relax liquidity constraints, but is costly because of the monitoring technology.

If the economy experiences binding constraints, $\mu^{SP} > \mu^{DE} > 1$, and the optimality condition implies that $\gamma^{SP} > \gamma^{DE} > 0$. A planner values liquidity more highly than decentralized agents and is therefore more willing than bankers to pay auditing costs to raise new equity. She internalizes that in doing so, she not only relaxes the constraint of the banker who obtains liquidity, but also pushes up the asset price at which all other bankers are fire-selling.

Proposition 8 *A social planner would sell a higher equity stake $\gamma^{SP} > \gamma^{DE}$ than decentralized bankers in states of binding constraints so as to mitigate financial amplification effects.*

Remark: The fundamental difference between fire sales and raising new equity in our example is that fire sales lead to aggregate price declines, which entail pecuniary externalities on other agents, whereas equity issuance entails private costs to bankers that do not have external effects.

6 Conclusions

Financial markets are inherently pro-cyclical – financing constraints endogenously loosen in good times and tighten in bad times, and this phenomenon can entail financial amplification effects: in case of a negative aggregate shocks, bankers experience binding borrowing constraints, which may require them to cut back on their economic activity by selling some of their asset holdings. This depresses asset prices, deteriorates their balance sheets, leads to tighter financing conditions, requires further fire sales etc.

This paper demonstrate that such financial amplification effects introduce an externality into the economy that leads individual bankers to undervalue liquidity in crisis states. Small agents take asset prices – and the tightness of financing conditions – as given and do not internalize the general equilibrium effects of their actions on prices and constraints. In particular, they do not internalize that fire sales during crises depress asset prices, which trigger amplification effects that hurt other bankers in the economy.

The undervaluation of liquidity in crisis times in turn leads bankers to take on excessive risk and buy insufficient insurance in their financing decisions, and to undervalue the benefits of raising new capital in crises. While we have limited our analysis to the financing decisions of bankers in the initial period, the externality would also lead to excessive real investment in projects that create exposure to systemic risk, as highlighted e.g. by Lorenzoni (2008).

Our paper develops a stylized model that allows us to analytically examine these inefficiencies and investigate related policy measure. In our model, liquidity shortages in period 1 lead to fire sales, but there is no debt carried from period 1 to period 2. There are two directions along which our setup of financial constraints could be extended. First, in infinite horizon models of financial amplification such as Kiyotaki and Moore (1997), falling asset price also reduce the value of collateral and lower the amount of debt that can be carried forward through a 'dynamic multiplier' effect. This is explored in Jeanne and Korinek (2010) for the case of uncontingent financial contracts. Second, as emphasized e.g. by Geanakoplos

(2009), changes in financial conditions are also reflected in endogenous changes in leverage. Both effects are likely to further strenghthen the externalities of financial amplification effects.

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A Mathematical Appendix

A.1 Bankers

Assumption A.1 *The parameters of the economy satisfy the following conditions:*

1. $\alpha t_1 \in (0, R_0^{\max}]$ *where the ceiling* $R_0^{\max} = R_0(\nu^{\max})$ *as defined in (18)*
2. $A^{\max} t_1 \leq s^{\max} = F'(t_1) t_1$

Part 1. of the assumption guarantees that the initial investment requirement αt_1 is low enough that it can be financed by borrowing from first generation households. Specifically, the assumption guarantees that the shadow price of liquidity ν lies within an interval $(0, \nu^{\max}]$ for which the optimal repayment function $b_1(\nu; A_1^{\omega})$ is defined for all $\omega \in \Omega$. For a larger initial investment requirement, bankers would run into one of the following problems: either $\nu > \bar{\nu}(A^{\min})$, which implies that they commit to repayments that they cannot meet in low states of nature even if they fire-sell their entire asset holdings; or $\nu > \hat{\nu}(A^{\max})$, which implies that their initial investment requirement is so large that they will have to fire-sell assets in all states of nature in period 1.

We could relax this assumption somewhat by imposing an explicit constraint on fire sales $f_1^{\omega} \leq t_1$ and by including equilibria with binding constraints across all states of nature in our analysis, but this would not affect the main results of our analysis.

Point 2. implies that the collateral of bankers in period 1 valued at the lowest possible fire-sale price is sufficient to back up their maximum financial promise $A^{\max} t_1$ in the highest state of nature in an equilibrium with partially binding constraints. It follows by implication that the collateral of bankers is sufficient to back up all equilibrium promises $\{b_1^{\omega}\}$ in all states of nature. The marginal product $F'(t_1)$ captures the marginal payoff of the asset for period 2, which represents all future periods of the economy, whereas $A^{\max}$ captures the maximum payoff for period 1 only. The condition is therefore likely to be satisfied in practice. If there were binding constraints on b_1^{ω} for some ω, we would have to explicitly account for this constraint in our analysis, but the main results of our paper would still hold.

A.2 First Generation Households

Assumption A.2 *The degree of relative risk aversion* $R(c)$ *of first generation households satisfies*

$$\frac{(A^{\max} + \bar{A}_2) t_1}{e + (A^{\max} + \bar{A}_2) t_1} \cdot R(c) < 1 \; \forall \; c$$

Note that $(A^{\max} + \bar{A}_2) t_1$ is an upper bound on what entrepreneurs can afford to repay to households in period 1 if they obtain the maximum possible output shock and fire-sell all of their assets at the maximum possible price $\bar{A}_2$; therefore the contracted repayment $b_1^{\omega} \leq (A^{\max} + \bar{A}_2) t_1$ and household consumption $c_1^{\omega} \leq e + (A^{\max} + \bar{A}_2) t_1 \; \forall \omega$.

The purpose of assumption A.2 is to ensure that the amount raised by Arrow securities is an increasing function of the promised repayment, i.e.

$$\begin{aligned}\frac{dm_1^\omega b_1^\omega}{db_1^\omega} &= m_1^\omega + b_1^\omega \cdot \frac{\partial m_1^\omega}{\partial b_1^\omega} = m_1^\omega + b_1^\omega \cdot \frac{u''(c_1^\omega)}{u'(c_0)} = \\ &= m_1^\omega \cdot \left[1 + \frac{b_1^\omega}{c_1^\omega} \cdot \frac{c_1^\omega u''\left(c_1^\omega\right)}{u'\left(c_1^\omega\right)}\right] = m_1^\omega \cdot \left[1 - \frac{b_1^\omega}{c_1^\omega} \cdot R(c_1^\omega)\right] > 0 \quad \text{by A.2}\end{aligned}$$

The assumption is satisfied if the product of relative risk aversion and debt is sufficiently low. For the standard value of relative risk aversion used in macroeconomics $R = 2$ the assumption holds as long as the debt repayment received by households in period 1 makes up less than half of their consumption. This is plausible since approximately two thirds of household income derives from wages.

If the assumption was violated, then the amount of finance raised $m_1^\omega b_1^\omega$ would fall as the promised repayment b_1^ω rises because the pricing kernel of consumers m_1^ω would fall faster than b_1^ω increases. In such a situation, the economy may be subject to multiple equilibria, since different amounts of promised repayments could lead to the same amount of finance raised.

A.3 Second Generation Households

Assumption A.3 *The production function of second generation households satisfies*

$$\eta_{qf} = -\frac{\partial q_1}{\partial f_1} \cdot \frac{f_1}{q_1} = -\frac{f_1 \cdot F''(f_1)}{F'(f)} < 1 \tag{A.1}$$

This assumption is satisfied for most regular production functions. It guarantees that the revenue raised by fire sales is an increasing function of the amount of assets sold, i.e.

$$\frac{dq_1 f_1}{df_1} = F'(f_1) + f_1 F''(f_f) > 0$$

If this assumption was violated, there would be multiple levels of fire sales that would raise a given amount of liquidity.

A.4 Proof of Lemma 2

The first part of lemma 2 holds since $q(\cdot) = \bar{A}_2$ and the left-hand side of (28) reduces to 1 if $\nu < \hat{\nu}(A_1)$, thereby coinciding with the decentralized optimality condition (15).

For the second part, define the auxiliary equation

$$\left(\frac{\bar{A}_2}{q(b_1 - A_1 t_1)} - \eta\right) \Big/ (1 - \eta) = \nu \cdot \frac{u'(e + b_1)}{u'(e - \alpha t_1)} \tag{A.2}$$

and denote $\tilde{b}_1(\nu; A_1, \eta)$ as the solution to this implicit equation in b_1. Observe that equation (A.2) reduces to the decentralized optimality condition (15) if we set $\eta = 0$ and to the planner's optimality condition (28) if we set $\eta = \eta_{qf}$. The function $\tilde{b}_1(A_1, \nu, \eta)$ is continuous in η for $\eta \leq 1$. Applying the implicit function theorem we find that $\partial\tilde{b}_1(\nu; A_1, \eta)/\partial\eta < 0$ for $\nu > \hat{\nu}(A_1)$ since $q(\cdot) < \bar{A}_2$ in that case. Fixing (ν, A_1) and going from $\eta = 0$ in (15) to $\eta = \eta_{qf} > 0$ in (28) therefore implies that the solutions to the implicit equations have to satisfy $b_1^{SP}(\nu; A_1) < b_1^{DE}(\nu; A_1)$. Similarly, we determine that $\partial^2\tilde{b}_1(\nu; A_1, \eta)/\partial\eta\partial A_1 < 0$ for $\nu > \hat{\nu}(A_1)$, which confirms the last part of the lemma.